MULTIGROUP ALBEDO THEORY WITH APPLICATION TO NEUTRONIC CALCULATIONS FOR A GAS CORE REACTOR

By

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Dedicated to My Parents

to Whom I Owe Everything.

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For nuclear reactors with effective reflectors, the balance of neutrons shows great sensitivity to the inventory of those neutrons which spend time in the reflector. It is essential, in developing a theory which is to explain neutronic phenomenon, such as the core nonleakage probabilities and the variation of the effective neutron multiplication factor, that the details of the transfer of neutrons across the core-reflector interface be included.

The main objective of this research is to develop a multigroup neutron albedo theory to neutronic calculations of thermal nuclear reactors. The neutron albedo theory method is an analytical and intuitive method that does not solve an explicit equation such as the Boltzmann transport equation. Rather, transport analysis relations are used to obtain information about the behavior of neutrons during segments of their lifetime, which are relatively easy to

visualize. The purpose is to obtain a variety of descriptive nonleakage probabilities and the effective and infinite neutron multiplication factors. Thus, the neutron albedo theory methodology is very different from the conventional methodology used in the XSDRNPM computer code.

The main parameters of the four-group albedo theory approach are investigated with the aid of calculational examples. The three reactor system models chosen are representative of one-dimensional, spherical geometry, and thermal reactor systems which exhibit transport effects to a significant degree. The computations are also performed on the systems with the XSDRNPM computer code to serve as a reference.

The most significant result is the ability of the four-group albedo method to compute the core nonleakage probabilities and the effective neutron multiplication factor with a reasonably fast and low-priced method that is clearly more understandable in physical terms than conventional neutron transport methods. This can be accomplished without calculating the core neutron fluxes of the reactor systems investigated. The most significant application value of the method is in the ability to calculate and understand reactivity coefficients.

The agreement of the theoretical calculations confirmed the validity of the multigroup neutron albedo theory for the nonleakage probability and effective neutron multiplication factor calculations for reactor system of the types treated.

CHAPTER I
INTRODUCTION

Background: Two-Group Albedo Theory

In 1958, a two-group albedo theory [1,2] was developed by Alan M. Jacobs for the calculation of reactivity temperature coefficients of nuclear reactors characterized by the Pennsylvania State University research reactor (PSR). The PSR is of the swimming pool variety with highly enriched fuel elements and natural water as moderator-coolant and reflector.

The agreement of the experimental measurements and theoretical calculations [1] confirmed the validity of this two-group albedo theory for temperature coefficient calculations for reactor systems of the type treated.

Motivation for the Work: Generalization to Multigroup Theory

For nuclear reactors with effective reflectors the balance of neutrons shows great sensitivity to the inventory of those neutrons which spend time in the reflector. It is essential, in developing a theory which is to explain neutronic phenomenon, such as the core nonleakage probabilities and the variation of the effective neutron multiplication factor, that the details of the transfer of neutron across the core-reflector interface be included.

The generalization of the two-group albedo theory to a multigroup albedo theory is required if the method of analysis is to be useful in systems that require a more detailed energy spectrum treatment.

Particular Application to a Gas Core Reactor

Gas core reactors have undergone theoretical and experimental studies at the University of Florida. A significant number of neutronic analyses have been conducted. The results of the researchers [3] indicated that gas core reactor concepts have some distinct advantages for power generation compared with the other nuclear power. It also has been demonstrated that typical neutronic calculations using multidimensional codes exhibit high computational cost. Because of the extremely important role of the reflector-moderator in gas core concepts, it is thought to be useful to apply the multigroup albedo approach to such systems. The gas core examples studied at the University of Florida provide excellent test cases for the value of this approach.

Dissertation Objectives

The main objective of this research is to develop a multigroup neutron albedo theory to neutronic calculations of thermal nuclear reactors. One major application is in the determination of the core nonleakage probabilities and

the variation of the effective neutron multiplication factor with a reasonably quick and inexpensive method.

Thus, the primary objective of this research can be summarized as follows:

(1) Develop a multigroup neutron albedo theory, and

(2) Compare the results of the calculations for the model problems based on this multigroup neutron albedo theory with the theoretical results obtained when performed with the XSDRNPM computer code [4].

Dissertation Organization

In Chapter II, the systematic derivation of the multigroup neutron albedo theory is presented.

The evaluation techniques for the core and reflector albedos are discussed in Chapter III.

In Chapter IV, the evaluation techniques for partial principal nonleakage probabilities, P_{o_i}, are derived and described.

In Chapters V, VI, and VII, the model problems are introduced and results obtained from static one-dimensional neutronic calculations.

The conclusions obtained from this research and some suggestions for future work are included in Chapter VIII.

The appendices provide detailed support for the material discussed in this work as well as detailed specification of some parameter calculations which are too cumbersome to include in the text body.

CHAPTER II
DERIVATION OF THE MULTIGROUP NEUTRON ALBEDO THEORY

Formulation of the Effective Neutron Multiplication Factor

All real reactors are to some degree nonuniform. The nonuniformity may consist of a reflector, i.e., a region of low absorption material around the core which serves to reflect neutrons back into the multiplying core-region, or it may consist of a breeding blanket which catches the escaping neutrons for the purpose of transmutation. For nuclear reactors with effective reflectors, the absorption rate of neutrons by the nuclei of the core shows enormous sensitivity to the inventory of those escaping (at least once) neutrons which spend time in the reflector.

To facilitate the present theory, it is convenient to express the effective neutron multiplication factor, k_{eff}, as the product of the infinite medium multiplication factor, k_{∞}, and the sum of two nonleakage probabilities, P_o, the total principal nonleakage probability, and P, the total secondary nonleakage probability. The first of these nonleakage probabilities, P_o, is defined as the fraction of neutrons which, having been produced by fission in the core, never traverse the core boundary. The second probability, P, is defined as the fraction of neutrons which, though

produced by fission in the core, spend time in the reflector but are eventually absorbed by nuclei of the core. Thus,

$$k_{eff} = k_{\infty} (P_o + P)$$

$$P_o = P_o (C, R) \tag{2-1}$$

$$P = P(P_o, \text{core and reflector albedos})$$

In Equation 2-1, C refers to core geometry and group constants, and R refers to reflector geometry and group constants.

Suppose there are m neutron energy groups indexed 1, 2,..., i,..., m, where 1 is the group of highest energy and m represents the thermal group. One can define k_{eff} as follows

$$k_{eff} = \sum_{i=1}^{m} P_{A_i}^{F} \frac{\nu_i \Sigma_{f_i}}{\Sigma_{a_i}^{F}} \tag{2-2}$$

where

$P_{A_i}^{F}$ is the fraction of fission neutrons in group i which are absorbed by fuel nuclei,

ν_i is the average number of fission neutrons released in a fission reaction induced by a neutron in group i,

Σ_{f_i} is the macroscopic fission cross section for group i, and

$\Sigma_{a_i}^{F}$ is the macroscopic absorption cross section of the fuel for group i.

Albedos

Neutrons undergo reflection and absorption when they are incident on a scattering/absorbing medium. The reflection coefficient, or albedo, is defined as the current of the reflected neutrons divided by the incident neutron current, i.e.,

$$\text{albedo} = \frac{J_{\text{reflected}}}{J_{\text{incident}}} \quad . \tag{2-3}$$

If the medium is of finite thickness, neutrons also are transmitted. The transmission coefficient, or transmittance, is similarly defined as the fraction of incident current that is transmitted through the medium

$$\text{transmittance} = \frac{J_{\text{transmitted}}}{J_{\text{incident}}} \quad . \tag{2-4}$$

For the purpose of simplifying the development, it is assumed here that there is but one thermal group so that neutron upscattering can be neglected. The reflector-core neutron transfer is described by means of m-group albedos of which there are $\frac{m(m+1)}{2}$ types under the neutron energy transfer assumptions of multigroup diffusion theory, specifically,

(1) The probability that a neutron from group 1 be reflected with group 1 energy, here denoted by ${}_1\alpha_1$ for core and ${}_1\beta_1$ for reflector.

(2) The probability that a neutron from group 1 be reflected with group 2 energy, here denoted by

$_1\alpha_2$ for core and $_1\beta_2$ for reflector.

... ...

(i) The probability that a neutron from group i be reflected with group i′ energy, here denoted by $_i\alpha_{i'}$ for core and $_i\beta_{i'}$ for reflector.

... ...

In matrix form, one can write the core (α) and reflector (β) albedos as follows

$$[\alpha] = \begin{bmatrix} {}_1\alpha_1 & {}_1\alpha_2 & {}_1\alpha_3 & \cdots & {}_1\alpha_i & \cdots & {}_1\alpha_m \\ 0 & {}_2\alpha_2 & {}_2\alpha_3 & \cdots & {}_2\alpha_i & \cdots & {}_2\alpha_m \\ 0 & 0 & {}_3\alpha_3 & \cdots & {}_3\alpha_i & \cdots & {}_3\alpha_m \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & {}_i\alpha_i & \cdots & {}_i\alpha_m \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 0 & \cdots & {}_m\alpha_m \end{bmatrix} \tag{2-5}$$

$$[\beta] = \begin{bmatrix} {}_1\beta_1 & {}_1\beta_2 & {}_1\beta_3 & \cdots & {}_1\beta_i & \cdots & {}_1\beta_m \\ 0 & {}_2\beta_2 & {}_2\beta_3 & \cdots & {}_2\beta_i & \cdots & {}_2\beta_m \\ 0 & 0 & {}_3\beta_3 & \cdots & {}_3\beta_i & \cdots & {}_3\beta_m \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & {}_i\beta_i & \cdots & {}_i\beta_m \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 0 & \cdots & {}_m\beta_m \end{bmatrix} \tag{2-6}$$

The neglect of upscattering leads to an upper triangular form for the albedo matrices.

Total Principal Nonleakage Probability, P_o

It is convenient to decompose P_o into its energy group contributions. Hence,

$$P_o = P_o\left(P_{o_1}, P_{o_2}, \ldots, P_{o_i}, \ldots, P_{o_m}\right) = 1 - \sum_{i=1}^{m} {}^{m}S_i \qquad (2\text{-}7)$$

where P_{o_i} is the probability that group i neutrons never spend time in the reflector and ${}^{m}S_i$ (henceforth termed the "source of initial leakage for group i neutrons") is defined as the fraction of group i neutrons that leaks from the core for the first time.

Source of Initial Leakage, ${}^{m}S_i$

The sources of initial leakage with the simplification of only two energy groups (i.e., m = 2) can be discussed with the aid of Figure 2-1.

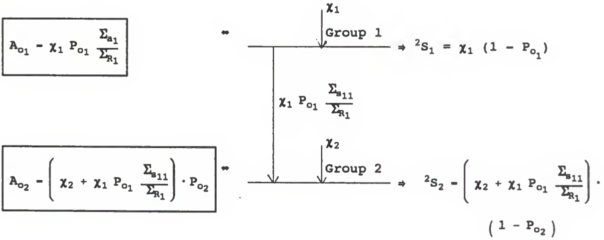


Figure 2-1. Schematic drawing showing the strategy for evaluation of the source of initial leakage under the two-group albedo approach.

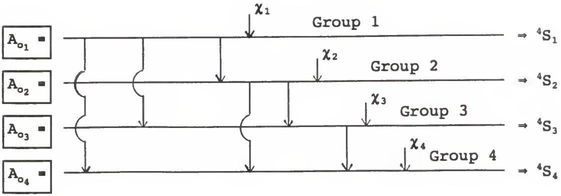
In Figure 2-1,

χ_i = the probability that a fission neutron will be born with an energy in group i. Note, $\chi_1 + \chi_2 = 1$.

A_{o_i} = the fraction of neutrons which, having been produced by fission in the core, never traverse the core boundary and are absorbed as group i neutron by nuclei of the core. Of course, $P_o - A_{o_1} + A_{o_2}$.

$\Sigma_{R_i} - \Sigma_{t_i} - \Sigma_{s_{ii}}$ = the group i macroscopic removal cross section for the core, Σ_{t_i} is the group i macroscopic total cross section for the core, and $\Sigma_{s_{ii}}$ is the macroscopic group-transfer cross section from group i to group i for the core.

A straightforward method to arrive at expressions for the multigroup source of initial leakage equations is to apply the concept of neutron balance to each energy group. An algorithm for calculating ${}^{m}S_i$ values is created employing the known source of initial leakage equations for group i of the (m-1) group albedo theory, ${}^{m-1}S_i$. Details of this idea are in Appendix A. For a small number of energy groups, $m \leq 10$, a heuristic schematic drawing is far more understandable and immediately useful. As an example, the ${}^{4}S_i$ and A_{o_i} equations are given in Figure 2-2 using the four-group albedo method. These results are then used in subsequent four-group calculations.



where

$$^4S_1 = \chi_1 \left(1 - P_{o_1}\right) \quad , \quad A_{o_1} = \chi_1 \, P_{o_1} \, \frac{\Sigma_{a_1}}{\Sigma_{R_1}} \; ,$$

$$^4S_2 = \left(\chi_2 + \chi_1 \, P_{o_1} \, \frac{\Sigma_{s12}}{\Sigma_{R_1}} \right) \left(1 - P_{o_2}\right) \; ,$$

$$A_{o_2} = \left(\chi_2 + \chi_1 \, P_{o_1} \, \frac{\Sigma_{s12}}{\Sigma_{R_1}} \right) P_{o_2} \, \frac{\Sigma_{a_2}}{\Sigma_{R_2}} \; ,$$

$$^4S_3 = \left[\chi_3 + \chi_1 \, P_{o_1} \, \frac{\Sigma_{s13}}{\Sigma_{R_1}} + \left(\chi_2 + \chi_1 \, P_{o_1} \, \frac{\Sigma_{s12}}{\Sigma_{R_1}} \right) P_{o_2} \, \frac{\Sigma_{s23}}{\Sigma_{R_2}} \right] \left(1 - P_{o_3}\right) \; ,$$

$$A_{o_3} = \left[\chi_3 + \chi_1 \, P_{o_1} \, \frac{\Sigma_{s13}}{\Sigma_{R_1}} + \left(\chi_2 + \chi_1 \, P_{o_1} \, \frac{\Sigma_{s12}}{\Sigma_{R_1}} \right) P_{o_2} \, \frac{\Sigma_{s23}}{\Sigma_{R_2}} \right] P_{o_3} \, \frac{\Sigma_{a_3}}{\Sigma_{R_3}} \; ,$$

$$^4S_4 = \left[\chi_4 + \chi_1 \, P_{o_1} \, \frac{\Sigma_{s14}}{\Sigma_{R_1}} + \frac{^4S_2}{1 - P_{o_2}} \, P_{o_2} \, \frac{\Sigma_{s24}}{\Sigma_{R_2}} + \frac{^4S_3}{1 - P_{o_3}} \, P_{o_3} \, \frac{\Sigma_{s34}}{\Sigma_{R_3}} \right] \left(1 - P_{o_4}\right)$$

$$A_{o_4} = \frac{^4S_4}{1 - P_{o_4}} \cdot P_{o_4}, \text{ and}$$

$$P_o = \sum_{i=1}^{4} A_{o_i}$$

Figure 2-2. Schematic drawing showing the evaluation of the source of initial leakage considering the four-group albedo approach.

Figure 2-2 shows one fission neutron, i.e., $\chi_1 + \chi_2 + \chi_3 + \chi_4 = 1$, cascading downward in energy from group to group as it is moderated by scattering collisions, a fraction being absorbed by the nuclei of the core, and a fraction leaving the core region for the first time.

The Ping-Pong Decision Process

Consider now the fraction of neutrons which do spend time in the reflector, but are absorbed eventually in the core. This total secondary nonleakage probability, P, is the sum of m separate processes, specifically,

(m) The probability that a neutron produced by fission in the core, leaks as thermal (i = m), and is absorbed in the core, here denoted by ${}^mP_m$.

... ...

(i) The probability that a neutron produced by fission in the core, leaks as a group i neutron, and is absorbed in the core, here denoted by ${}^mP_i$.

... ...

(1) The probability that a neutron produced by fission in the core, leaks as a group 1 neutron, and is absorbed in the core, here denoted by ${}^mP_1$.

In these terms,

$$P = \sum_{i=1}^{m} {}^mP_i \qquad (2\text{-}8)$$

where the ${}^{m}P_i$ are termed "the effective partial secondary nonleakage probability."

It is convenient to decompose ${}^{m}P_i$ into its energy group contributions. Hence

$$ {}^{m}P_i = {}^{m}S_i \sum_{i'=i}^{m} {}^{m}_{i}Q_{i'} , \tag{2-9} $$

where

${}^{m}_{i}Q_{i'}$, termed the fractional secondary nonleakage probability, is the probability that if one neutron leaks as a group i neutron from the core, and returns as a group i' neutron, it will be absorbed in the core. The partial secondary nonleakage probability, ${}^{m}Q_i$, is defined by

$$ {}^{m}Q_i = \sum_{i'=i}^{m} {}^{m}_{i}Q_{i'} . \tag{2-10} $$

Algorithm for ${}^{m}P_i$ $(m \geq 3)$

The following algorithm for calculating ${}^{m}Q_i$ is developed in a manner such that assumed known partial secondary nonleakage probabilities of m-1 group albedo theory imply results for m group theory. The starting expressions are from the two group albedo theory [2] and given by

$$ {}^{2}Q_1 = \frac{\left(1 - {}_1\alpha_1\right)\ {}_1\beta_1}{1 - {}_1\alpha_1\ {}_1\beta_1} - \frac{{}_1\beta_1\ {}_1\alpha_2}{1 - {}_1\alpha_1\ {}_1\beta_1} \cdot \frac{\left(1 - {}_2\beta_2\right)}{1 - {}_2\alpha_2\ {}_2\beta_2} + $$

$$ \frac{{}_1\beta_2}{1 - {}_1\alpha_1\ {}_1\beta_1} \cdot \frac{\left(1 - {}_2\alpha_2\right)}{1 - {}_2\alpha_2\ {}_2\beta_2} \quad \text{and} \tag{2-11} $$

$$ {}^{2}Q_2 = \frac{\left(1 - {}_2\alpha_2\right)\ {}_2\beta_2}{1 - {}_2\alpha_2\ {}_2\beta_2} \ . $$

Derivation of Equation 2-11 constitutes Appendix B.

First step: Evaluation of ${}^{m}P_{m}$

Using the assumed known expression of ${}^{m-1}Q_{m-1}$, replace the subscript (m-1) by the subscript m; thus,

$$ {}^{m}P_{m} = {}^{m}S_{m}\ {}^{m}Q_{m} = {}^{m}S_{m}\ {}^{m-1}Q_{m-1}\ (m-1 \rightarrow m) \tag{2-12} $$

or

$$ {}^{m}P_{m} = {}^{m}S_{m}\ \frac{\left(1 - {}_m\alpha_m\right)\ {}_m\beta_m}{1 - {}_m\alpha_m\ {}_m\beta_m} \ . $$

Second step: Evaluation of ${}^{m}P_{i}$ $(2 \le i \le m-1)$

$$ {}^{m}P_{i} = {}^{m}S_{i}\ {}^{m}Q_{i} = {}^{m}S_{i}\ {}^{m-1}Q_{i-1} \begin{pmatrix} i-1 \rightarrow i \\ \cdots \quad \cdots \\ \cdots \quad \cdots \\ m-1 \rightarrow m \end{pmatrix} . \tag{2-13} $$

Last Step: Evaluation of ${}^{m}P_{1}$

For the evaluation of ${}^{m}P_{1}$, one can consider m separate processes given by

$$ {}^{m}P_{1} = {}^{m}S_{1} \left[{}_{1}^{m}Q_{m} + {}_{1}^{m}Q_{m-1} + \cdots + {}_{1}^{m}Q_{2} + {}_{1}^{m}Q_{1} \right] . \tag{2-14} $$

Keeping in mind that ${}^{m}Q_{i}$ is the fraction that is absorbed in the core if one neutron leaks as a group i neutron from the core, one can define ${}^{m}\overline{Q}_{i}$ as the fraction that is absorbed in

the reflector if one neutron leaks as a group i neutron from the reflector.

For computer implementation one can write

$$ {}^{m}\overline{Q}_{i} = {}^{m}Q_{i}\begin{pmatrix}\alpha \to \beta \\ \beta \to \alpha\end{pmatrix} . \tag{2-15}$$

<u>First process: Evaluation of ${}_{1}^{m}Q_{m}$</u>

$$ {}_{1}^{m}Q_{m} = \frac{{}_{1}\beta_{m}}{1 - {}_{1}\alpha_{1}\ {}_{1}\beta_{1}} \left[1 - {}^{m-1}Q_{m-1}\,(m - 1 \to m) \right] . \tag{2-16}$$

<u>Second process: Evaluation of ${}_{1}^{m}Q_{m-1}$</u>

$$ {}_{1}^{m}Q_{m-1} = \frac{{}_{1}\beta_{m-1}}{1 - {}_{1}\alpha_{1}\ {}_{1}\beta_{1}} \left[1 - {}^{m-1}\overline{Q}_{m-2}\begin{pmatrix} m-2 & \to & m-1 \\ m-1 & \to & m\end{pmatrix}\right] . \tag{2-17}$$

<u>(m-1)th process: Evaluation of ${}_{1}^{m}Q_{2}$</u>

$$ {}_{1}^{m}Q_{2} = \frac{{}_{1}\beta_{2}}{1 - {}_{1}\alpha_{1}\ {}_{1}\beta_{1}} \left[1 - {}^{m-1}\overline{Q}_{1}\begin{pmatrix} 1 & \to & 2 \\ 2 & \to & 3 \\ \cdots & \cdots & \cdots \\ m-1 & \to & m\end{pmatrix}\right] . \tag{2-18}$$

<u>(m)th process: Evaluation of ${}_{1}^{m}Q_{1}$</u>

$$ {}_{1}^{m}Q_{1} = \frac{\left(1 - {}_{1}\alpha_{1}\right)\ {}_{1}\beta_{1}}{1 - {}_{1}\alpha_{1}\ {}_{1}\beta_{1}} - \frac{{}_{1}\beta_{1}}{1 - {}_{1}\alpha_{1}\ {}_{1}\beta_{1}} \sum_{i=2}^{m} {}_{1}\alpha_{i}\left[1 - {}^{m}Q_{i}\right] . \tag{2-19}$$

Multigroup Neutron Albedo Approach Flow Diagram

As the multigroup albedo approach becomes more complex, a flow diagram is helpful in planning and structuring a program for solving a calculational example. The basic features of the multigroup neutron albedo strategy are shown in the simplified flow diagram of Figure 2-3. The actual programs are contained in Appendix G.

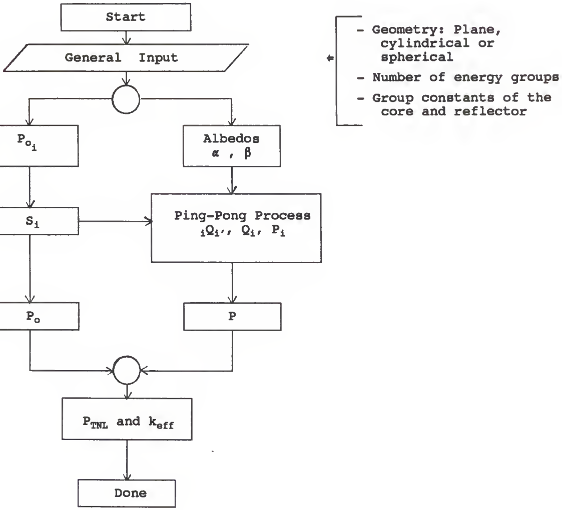


Figure 2-3. Simplified flow diagram of multigroup neutron albedo strategy.

CHAPTER III
EVALUATION TECHNIQUES FOR THE CORE (α) AND REFLECTOR (β) ALBEDOS

Introduction

Neutrons undergo reflection when they are incident on a scattering/absorbing medium. The reflection coefficient, or albedo, is defined as the current of the reflected neutrons divided by the incident neutron current. If the medium is of finite thickness, neutrons are also transmitted. The transmission coefficient, or transmittance, is similarly defined as the fraction of incident current that is transmitted through the medium. Here the evaluation techniques for the albedo/transmittance values are confined to two different approaches: (1) low-order approximations to solution of the Boltzmann transport equation such as diffusion theory, and (2) first interaction free-flight transport methods.

Albedo Calculation Using the Neutron Diffusion Approximation

The diffusion approximation to the Boltzmann transport equation is sufficiently accurate for the calculation of neutron densities in systems which are homogeneous, or nearly homogeneous, and which are so large that the radius of curvature of their boundaries is almost everywhere large

compared to the mean free path of the neutrons. These conditions are satisfied in large solid or liquid reactors of the usual, rather regular, shapes.

In order to demonstrate the considerations involved, the case of a spherical core with an infinite reflector and example albedo calculations for ${}_1\alpha_4$ and ${}_2\beta_3$ are here outlined.

Determination of ${}_1\alpha_4$

The governing equations and boundary conditions for the determination of ${}_1\alpha_4$ are (see Figure 3-1)

$$\begin{aligned}
&- D_1\nabla^2\phi_1 + \Sigma_{R_1}\phi_1 = 0 \\
&- \Sigma_{S_{12}}\phi_1 - D_2\nabla^2\phi_2 + \Sigma_{R_2}\phi_2 = 0 \\
&- \Sigma_{S_{13}}\phi_1 - \Sigma_{S_{23}}\phi_2 - D_3\nabla^2\phi_3 + \Sigma_{R_3}\phi_3 = 0 \\
&- \Sigma_{S_{14}}\phi_1 - \Sigma_{S_{24}}\phi_2 - \Sigma_{S_{34}}\phi_3 - D_4\nabla^2\phi_4 + \Sigma_{R_4}\phi_4 = 0
\end{aligned} \qquad (3\text{-}1)$$

At $r = 0$, ϕ_1, ϕ_2, ϕ_3, and ϕ_4 are finite.

At $r = R$, $J_-]_{i=1} = 1$, $J_-]_{i=2,3,4} = 0$

where $J_\pm$ are the partial current densities and all group parameters $\left(D_i \text{ , } \Sigma_{R_i} \text{ , and } \Sigma_{S_{ii'}} \right)$ refer to the homogeneous core. At $r = R$, ${}_1\alpha_4 = \dfrac{J_+]_{i=4}}{J_-]_{i=1}} = \dfrac{\phi_4}{4} - \dfrac{D_4}{2}\dfrac{d}{dr}\phi_4$. The results of this calculation as well as all other four-group albedos used in this work are presented in Appendices C and D.

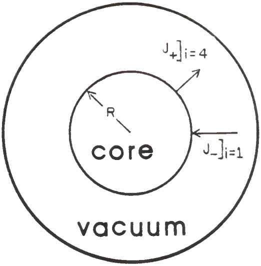


Figure 3-1. Schematic drawing showing the evaluation of ${}_1\alpha_4$.

Determination of ${}_2\beta_3$

The governing equations and boundary conditions for determination of ${}_2\beta_3$ are (see Figure 3-2)

$$- D_2\nabla^2\phi_2 + \Sigma_{R_2}\phi_2 = 0$$

(3-2)

$$- \Sigma_{S_{23}}\phi_2 - D_3\nabla^2\phi_3 + \Sigma_{R_3}\phi_3 = 0$$

As $r \to \infty$, ϕ_2 and ϕ_3 are finite.

At $r=R$, $J_+]_{i=2} = 1$, $J_+]_{i=3} = 0$

where $J_\pm$ are the partial current densities and all group parameters $\left(D_i , \Sigma_{R_i} , \Sigma_{S_{ii'}} \right)$ refer to the homogeneous reflector. At $r = R$, ${}_2\beta_3 = \dfrac{J_-]_{i=3}}{J_+]_{i=2}} = \dfrac{\phi_3}{4} + \dfrac{D_3}{2}\dfrac{d}{dr}\phi_3$.

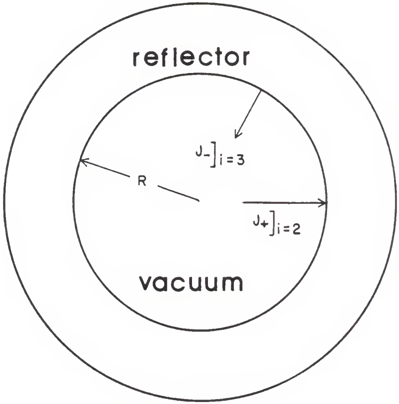


Figure 3-2. Schematic drawing showing the evaluation of ${}_2\beta_3$.

First-Interaction Transmission Coefficients

If the radius of curvature of boundaries is small compared to the mean free path of neutrons such as is the case for gas core reactors, core albedo considerations are changed to core transmittance considerations. Transmission coefficients can be evaluated following a suggestion by Tsoulfanidis [5] as follows: Consider a point isotropic monoenergetic source to be on the surface of a spherical core of diameter 2R (see Figure 3-3). For neutrons emitted from the reflector at an angle ω, measured from the axis or diameter of the spherical core, the probability of interaction in the core is 1-exp $[-\Sigma_t \times 2R \times \cos\omega]$. The probability of emission between angles ω and $\omega + d\omega$ is $\frac{\sin\omega}{2} d\omega$. Σ_t is the core total macroscopic cross section. The core transmittance, γ , is given by

$$\gamma = \frac{\int_{\omega=0}^{\pi/2} \exp\left[-\Sigma_t \times 2R \times \cos\omega \right] \frac{\sin\omega}{2} \, d\omega}{\int_{\omega=0}^{\pi/2} \frac{\sin\omega}{2} \, d\omega} \tag{3-3}$$

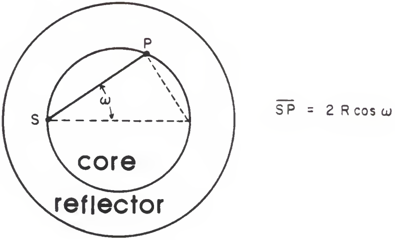


Figure 3-3. Schematic drawing for evaluation of the core transmission coefficients due to a point isotropic neutron source on a spherical core.

Other Approaches for Albedo/Transmittance Evaluation

Here are presented additional options for albedo and transmittance calculation. Analytical studies of the albedo equations in the theory of the diffusion and slowing down of neutrons were carried out by Orlov [6] in 1961. When the invariant imbedding theory [7, 8] is applied to the Boltzmann equation of neutron transport some of quantities of interest become the reflection function and the transmission function. The mathematical theory of the invariant imbedding approach has been studied by Wing [9] especially for the reflection probability.

Finally, in this work, the core and reflector albedo values are obtained employing the multigroup neutron diffusion theory and the core transmittance results are evaluated using the first-interaction, free-flight transport method.

CHAPTER IV
EVALUATION TECHNIQUES FOR THE PARTIAL PRINCIPAL NONLEAKAGE PROBABILITY, P_{oi}

Introduction

The initial nonleakage probability, P_{oi}, is the probability that neutrons from group i never spend time in the reflector. The analytical expression of P_{oi} in terms of the core and reflector parameters involve analytical complications. Here, two evaluation approaches for the initial nonleakage probabilities are suggested: (1) a bare core approximation, and (2) a consistent multigroup diffusion theory method.

Basic Ideas

As an introductory, heuristic example, consider the case of one-group analysis in an infinite slab core reflected by infinitely thick media. The one-group neutron flux in the core varies as cos (Bx), where B^2 is the buckling of the core material. Therefore, $\nu\Sigma_f$ cos (Bx) is the source function which should be placed in a convolution with a nonreentrant flux kernel in order to determine the neutron flux which has never been in the reflector. The

kernel, K(x), for this component of the flux satisfies the differential equation

$$D \frac{d^2K}{dx^2} - \Sigma_a K = 0 \tag{4-1}$$

and has the boundary conditions that K(x) be continuous and that the resultant reentrant current of neutrons to the core at the core-reflector interfaces be zero. The second boundary condition is approximated by requiring K(x) to vanish at the extrapolated boundaries of the core, i.e., $K(x = \tilde{a}) = 0$ where $\tilde{a} = a + 2.1312$ D and a is the position of the core-reflector interface (see Figure 4-1).

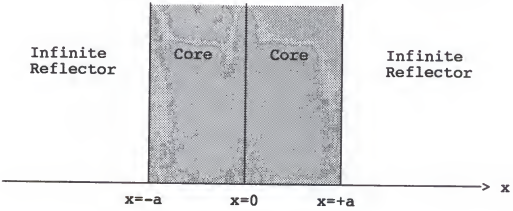


Figure 4-1. Reflected slab reactor.

With a unit strength source plane at $x = x_o$, and the boundary conditions

$$\left[D \frac{d K_-}{dx} - D \frac{d K_+}{dx}\right]_{x=x_o} = 1 ,$$

$$\left[K_+ \right]_{x=x_o} = \left[K_- \right]_{x=x_o} \text{, and} \tag{4-2}$$

$$\left[K_+ \right]_{x=\tilde{a}} = 0 = \left[K_- \right]_{x=-\tilde{a}}$$

the solution for K(x) is

$$K_- = K(x) = \frac{\sinh[c_1 (\tilde{a} + x)]}{c_1 D \sinh[c_1 (\tilde{a} + x_o)] [\coth[c_1 (\tilde{a} - x_o)] + \coth[c_1 (\tilde{a} + x_o)]]}$$

$$\text{for } -a \le x < x_o \tag{4-3}$$

$$K_+ = K(x) = \frac{\sinh[c_1 (\tilde{a} - x)]}{c_1 D \sinh[c_1 (\tilde{a} - x_o)] [\coth[c_1 (\tilde{a} - x_o)] + \coth[c_1 (\tilde{a} + x_o)]]}$$

$$\text{for } x_o < x \le a$$

where

$$c_1 = \sqrt{\frac{\Sigma_a}{D}} .$$

Thus, the solution for the flux component which has never been in the reflector, denoted here by ϕ_o, is

$$\phi_o\ (x) = \int_{x_o=-a}^{a} \nu\ \Sigma_f \cos\ (Bx_o)\ K\ (x,\ x_o)\ d\ x_o\ . \tag{4-4}$$

Under the one-group assumption, the initial nonleakage probability, P_{o_1}, is given in terms of ϕ_o, as

$$P_{o_1} = \left[1 + \frac{J_{+o}\ (x=a)}{\Sigma_a \int_{x=o}^{a} \phi_o\ dx} \right]^{-1} \tag{4-5}$$

Notice that it is the ratio of the outgoing neutron current at the core boundary to the absorption rate of the flux which has not traversed the core boundary which is of importance.

This one-group treatment is extended to a multigroup evaluation of P_{o_i} later in this chapter.

Bare Core Approximation

A simplification suggested by Jacobs [1] is to use for P_{o_i} the initial nonleakage probabilities which would be applicable to a bare core reactor with the dimensions of the core of the reflected reactor. One can write

$$P_{o_i} = \left[1 + (B')^2\ \frac{D_i}{\Sigma_{R_i}} \right]^{-1} \tag{4-6}$$

where $(B')^2$ is the geometrical buckling of a bare core of the same dimensions as the reflected core, Σ_{R_i} are the core macroscopic removal cross sections, and D_i are the core diffusion coefficients.

Consistent Multigroup Diffusion Theory Method

The initial nonleakage probabilities, P_{o_i}, are given by

$$P_{o_i} = \left[1 + B_i^2 \frac{D_i}{\Sigma_{R_i}} \right]^{-1} \tag{4-7}$$

under the multigroup diffusion approximation. In Equation (4-7),

$$B_i^2 = - \frac{\int_S \nabla \phi_{o_i} \cdot d\,S}{\int_V \phi_{o_i}\, d\,V}$$

$$\Sigma_{R_i} = \Sigma_{t_i} - \Sigma_{s_{ii}}$$

Σ_{R_i} are the core macroscopic removal cross sections, Σ_{t_i} are the core total macroscopic cross sections, $\Sigma_{s_{ii}}$ are the core self-group-transfer cross sections, D_i are the core diffusion coefficients, ϕ_{o_i} is the component of the i group neutron flux which has not been in the reflector, and the

integrations are over the volume and the surface of the reactor core.

In spherical geometry (i.e., a spherical core in an infinite reflector) the ϕ_{o_i} are given by

$$\phi_{o_i}(r) = \phi_i(r) - \frac{A_i}{r} \sinh\left(\sqrt{\frac{\Sigma_{R_i}}{D_i}}\; r\right) \tag{4-8}$$

and subject to the boundary conditions at r = R, $J_{o_{i-}} = 0$. The i group neutron flux, $\phi_i(r)$, includes all neutrons whether or not they have spent time in the reflector.

Because of the algebraicly complicated developments involved, solution of $\phi_{o_i}(r)$ and P_{o_i} is relegated to Appendix E. The results listed there are used in the remainder of this work.

CHAPTER V
STATIC, ONE-DIMENSIONAL NEUTRONIC ANALYSIS OF A LARGE OPTICAL-PATH-LENGTH CORE REACTOR (TWO-REGION SOLID CORE REACTOR)

Introduction

In this section, the main parameters of the four-group albedo theory are investigated with the aid of a calculational example. A solid core reactor with a water moderator/reflector is studied as a first model case. This problem is selected because both S_n transport theory and diffusion theory approximations can be successfully applied and give essentially the same results [10].

Core Atomic Densities

As a first example problem consider a large optical-path-length spherical core reactor surrounded by an infinite water reflector where it is assumed that the core materials are a homogeneous mixture. In Table 5-1 are the core atomic densities employed.

TABLE 5-1. CORE ATOMIC DENSITIES OF THE SOLID CORE REACTOR PROBLEM

Material	Atomic Densities (atoms/(barn x cm))
Uranium - 235	.12200 E-03
Uranium - 238	.59700 E-02
Oxygen - 16	.34420 E-01
Natural Chromium	.93460 E-03
Manganese - 55	.94200 E-04
Natural Iron	.33470 E-02
Natural Nickel	.47110 E-03
Hydrogen - 1	.44470 E-01

Core and Reflector Group Constants

For this particular problem, NITAWL [11] is used to generate a 123 fine group neutron cross-section library. The 123 fine group neutron cross-sections (30 thermal groups) are collapsed down to a 26 energy-group structure (6 thermal groups) and then to a 4 energy-group structure (one thermal group) with XSDRNPM. Tables 5-2 and 5-3 show the core and reflector group constants, respectively.

TABLE 5-2. CORE GROUP CONSTANTS OF THE SOLID CORE REACTOR PROBLEM FROM THE XSDRNPM CODE

	Four Energy-Group Structure			
	i = 1	i = 2	i = 3	i = 4
D	.17607 E+01	.80339	.47001	.19923
$\Sigma_{n,2n}$	.13954 E-03	.00000	.00000	.00000
χ	.74415	.25565	.20189 E-03	.12480 E-08
Σ_a	.33928 E-02	.18935 E-02	.17635 E-01	.57172 E-01
$\nu\Sigma_f$	.72250 E-02	.51635 E-02	.59613 E-02	.66730 E-01
Σ_t	.27502	.71374	.13068 E+01	.22259 E+01

	$\Sigma_{s_{ii'}}$		
i → i'	i' = 2	i' = 3	i' = 4
1	.89651 E-01	.46418 E-03	.15529 E-06
2	-	.95330 E-01	.31330 E-04
3	-	-	.98090 E-01

- (D in cm and Σ in cm^{-1})
- Radius of the spherical core = 64.0 cm.

From this table, the ratio of core diameter to the total mean free paths in the solid core is found to be very large (~285.0 for thermal neutrons), thus, this solid core has indeed very large optical dimensions.

TABLE 5-3. REFLECTOR GROUP CONSTANTS OF THE SOLID CORE REACTOR PROBLEM FROM THE XSDRNPM CODE

	Four Energy-Group Structure			
	i = 1	i = 2	i = 3	i = 4
D	.18109 E+01	.78453	.50770	.14915
Σ_a	.31290 E-03	.95302 E-05	.57242 E-03	.15539 E-01
Σ_t	.27073	.85161	.15510 E+01	.30612 E+01

	$\Sigma_{s_{ii'}}$		
i → i'	i' = 2	i' = 3	i' = 4
1	.11270	.69381 E-03	.23278 E-06
2	-	.14163	.46992 E-04
3	-	-	.14601

- (D in cm and Σ in cm^{-1})

Core (α) and Reflector (β) Albedos

The diffusion theory approximation within the solid core reactor is valid. Thus, Tables 5-4 and 5-5 show the core and reflector albedo values obtained using the expressions in Appendices C and D, respectively.

TABLE 5-4. CORE ALBEDOS OF THE SOLID CORE REACTOR PROBLEM USING THE FOUR-GROUP ALBEDO THEORY

	${}_i\alpha_{i'}$			
i → i'	i' = 1	i' = 2	i' = 3	i' = 4
1	.13824	.45412	.95613 E-01	.10531
2	-	.30383	.20285	.87856 E-01
3	-	-	.37764	.21696
4	-	-	-	.65669

- Radius of the spherical core = 64.0 cm.

TABLE 5-5. REFLECTOR ALBEDOS OF THE SOLID CORE REACTOR PROBLEM USING THE FOUR-GROUP ALBEDO THEORY

$i \rightarrow i'$	${}_i\beta_{i'}$			
	$i' = 1$	$i' = 2$	$i' = 3$	$i' = 4$
1	.18267 E-01	.20087	.90822 E-01	.14104
2	-	.18251	.22292	.21026
3	-	-	.28086	.39499
4	-	-	-	.81662

- Radius of the spherical core = 64.0 cm.
- Infinite water reflector.

From this table, the probability that a neutron from group 1 be reflected by the reflector is found to be small (${}_1\beta_1 + {}_1\beta_2 + {}_1\beta_3 + {}_1\beta_4 \approx .45$). Thus, 55% of the incident neutrons from group 1 are absorbed in the reflector.

Partial Principal Nonleakage Probabilities, P_{o_i}

The evaluation of the partial principal nonleakage probabilities constitute the key item for a multigroup albedo theory technique. The diffusion theory approximation within the solid core reactor is valid. In Table 5-6 are the partial principal nonleakage probability, P_{o_i}, and source of initial leakage, S_i, values obtained using the consistent multigroup diffusion theory method (which should be valid). Values are obtained using the expressions summarized in Appendix E.

TABLE 5-6. PARTIAL PRINCIPAL NONLEAKAGE PROBABILITIES, P_{o_i}, AND SOURCES OF INITIAL LEAKAGE, S_i, OF THE SOLID CORE PROBLEM USING THE FOUR-GROUP ALBEDO THEORY METHOD

	Four-Group Albedo Theory Method			
	i = 1	i = 2	i = 3	i = 4
P_{o_i}	.94055	.97792	.98077	.96782
S_i	.44242 E-01	.20447 E-01	.17131 E-01	.23846 E-01

- Radius of the spherical core = 64.0 cm.
- Infinite water reflector.

From this table, the partial principal nonleakage probabilities, P_{o_i}, are very large and the sources of initial leakage, S_i, are very small. Thus, this solid core reactor problem shows large total principal nonleakage probability, P_o, i.e., $P_o = 1 -(S_1 + S_2 + S_3 + S_4) = .89433$. In other words, 89.43% of the neutrons which are produced by fission in the core never traverse the core boundary.

The Ping-Pong Decision Process

Considering now the fraction of neutrons which do spend time in the reflector, in Table 5-7 are the fractional secondary nonleakage probability, ${}_iQ_{i'}$, values obtained using the ping-pong decision process developed in Chapter II. The program ALB1 used for these numerical evaluations is in Appendix G.

TABLE 5-7. FRACTIONAL SECONDARY NONLEAKAGE PROBABILITIES, ${}_iQ_{i'}$, OF THE SOLID CORE PROBLEM USING THE PING-PONG DECISION PROCESS

$i \rightarrow i'$	${}_iQ_{i'}$			
	$i' = 1$	$i' = 2$	$i' = 3$	$i' = 4$
1	.95088 E-02	.13974	.65900 E-01	.10468
2	-	.10804	.17081	.16480
3	-	-	.16858	.32712
4	-	-	-	.60456

- Radius of the spherical core = 64.0 cm.
- Infinite water reflector.

From Tables 5-6 and 5-7, the total secondary nonleakage probability, P, is given by

$$P = \sum_{i=1}^{4} \left[S_i \sum_{i'=i}^{4} {}_iQ_{i'} \right] = .46129\ E\text{-}01 \ .$$

Thus, 4.61% of the neutrons which are produced by fission in the core and spend time in the reflector are eventually absorbed by nuclei of the core.

Dissection of P_o and P

It is interesting to know (1) the fraction of neutrons produced by fission in the core, which never traverse the core boundary and are absorbed as group i, here denoted by A_{o_i}, and (2) the fraction of neutrons which are produced by fission in the core, spend time in the reflector and are eventually absorbed by nuclei of the core as group i, here denoted by A_i.

In Table 5-8, are the probabilities, A_{o_i} and A_i, where

$$P_o = \sum_{i=1}^{4} A_{o_i} \quad \text{and} \quad P = \sum_{i=1}^{4} A_i \ .$$

The numerical values are found using the program ALB1 described in Appendix G.

TABLE 5-8. NONLEAKAGE PROBABILITIES, A_{o_i} AND A_i, OF THE SOLID CORE REACTOR PROBLEM USING THE FOUR-GROUP ALBEDO THEORY METHOD

	Four-Group Albedo Theory Method			
	i = 1	i = 2	i = 3	i = 4
A_{o_i}	.26402 E-01	.17628 E-01	.13317	.71714
A_i	.63172 E-05	.10933 E-03	.18880 E-02	.44126 E-01

From Table 5-8, the total nonleakage probability, P_{TNL}, is found to be large, approximately 94%, i.e.,

$$P_{TNL} = \sum_{i=1}^{4} \left(A_{o_i} + A_i \right) = .94046 \ .$$

About 94% of the neutrons produced by fission in the core are absorbed by nuclei of the core.

Effective Neutron Multiplication Factor, k_{eff}

The neutrons produced by the (n,2n) interaction in the solid core are accounted for in the k_{eff} evaluation. Thus, k_{eff} is given by

$$k_{eff} = \left(A_{o_1} + A_1 \right) \frac{\nu_1 \Sigma_{f_1} + 2\ \Sigma_{n,2n}}{\Sigma_{a_1}} + \sum_{i=2}^{4} \left(A_{o_i} + A_i \right) \frac{\nu_i\ \Sigma_{f_i}}{\Sigma_{a_i}} \ , \qquad (5\text{-}1)$$

i.e., k_{eff} = .99512.

Comparison: XSDRNPM Versus Albedo Method Results

A one-dimensional static neutronic analysis has been performed on the solid core reactor problem with the XSDRNPM computer code. The $(A_{o_i} + A_i)$ results performed with XSDRNPM and albedo method are compared with one another in Table 5-9.

TABLE 5-9. COMPARISON OF $(A_{o_i} + A_i)$ RESULTS OF THE SOLID CORE REACTOR PROBLEM USING THE XSDRNPM COMPUTER CODE AND THE ALBEDO METHOD

	$(A_{o_i} + A_i)$			
	i = 1	i = 2	i = 3	i = 4
XSDRNPM	.23226 E-01	.17755 E-01	.13548	.76360
Albedo	.26409 E-01	.17738 E-01	.13505	.76126

- Radius of the spherical core = 64.0 cm
- Infinite water reflector for the albedo results
- Reflector 100 cm thick for XSDRNPM, S_4 P_3, calculation
- XSDRNPM parameters: Four-group, S_4 P_3, number of spacial intervals = 60, overall convergence = 1.0 E-04
- XSDRNPM calculation on an IBM 3090-600J

Note that in Table 5-9 the values of $(A_{o_i} + A_i)$ are compared because the XSDRNPM computer code does not give A_{o_i} and A_i values separably as does the albedo method.

In Table 5-10 are the total nonleakage probability, P_{TNL}, and effective neutron multiplication factor, k_{eff}, for the system as obtained from XSDRNPM, four-group, S_4 P_3 calculation and the four-group albedo method.

TABLE 5-10. TOTAL NONLEAKAGE PROBABILITY, P_{TNL}, AND EFFECTIVE NEUTRON MULTIPLICATION FACTOR, k_{eff}, VALUES OF THE SOLID CORE REACTOR PROBLEM USING THE XSDRNPM COMPUTER CODE AND THE ALBEDO METHOD

	XSDRNPM	Albedo Method
P_o	-	.89433
P	-	.46129 E-01
$P_{TNL} = P_o + P$	.94256	.94046
k_{eff}	.99667	.99512
CPU Time	8.0 sec	< 1 sec

- Radius of the spherical core = 64.0 cm
- Infinite water reflector for the albedo results
- Reflector 100 cm thick for XSDRNPM calculation
- XSDRNPM parameters: Four-group, S_4 P_3, number of spacial intervals = 60, overall convergence = 1.0 E-04
- XSDRNPM calculation on an IBM 3090-600J

In order to avoid relying on a mainframe system, a personal computer version of the four-group albedo approach program is created and written in FORTRAN-77 (see Appendix G). Table 5-10 shows that, for the examined configuration, the albedo theory approximation is found to underpredict k_{eff} by about 0.2% relative to the S_4 P_3 result.

For various core density factors, the k_{eff} results performed with XSDRNPM are compared with albedo method in Table 5-11. These density factors are used to effect a density variation in a mixture. Zero for a density factor affords a convenient way for mocking a void region.

TABLE 5-11. COMPARISON OF k_{eff} RESULTS OF THE SOLID CORE REACTOR PROBLEM USING VARIOUS CORE DENSITY FACTORS (D.F.) FROM THE XSDRNPM CODE AND THE FOUR-GROUP ALBEDO METHOD

D.F.	k_{eff}		k_{eff}	CPU Time	
	XSDRNPM	Albedo	Relative Error	XSDRNPM	Albedo
1.2	1.0133	1.0120	.128 E-02	9.7 sec	< 1 sec
1.1	1.0059	1.0044	.149 E-02	8.8 sec	< 1 sec
1.0	.99667	.99512	.155 E-02	8.0 sec	< 1 sec

- Radius of the spherical core = 64.0 cm
- Infinite water reflector for the albedo results
- Reflector 100 cm thick for XSDRNPM calculation
- XSDRNPM parameters: Four-group, S_4 P_3, number of spacial intervals = 60, overall convergence = 1.0 E-04
- XSDRNPM calculation on an IBM 3090-600J

- k_{eff} relative error = $\dfrac{k_{eff}\,(\text{XSDRNPM}) - k_{eff}\,(\text{albedo})}{k_{eff}\,(\text{XSDRNPM})}$

Table 5-11 shows that for the density factors used, the albedo theory approximation is found to underpredict k_{eff} by about 0.2% relative to the S_4 P_3 results. An examination of the calculation times, in this table, shows that albedo method is much faster than the S_4 P_3 approximation.

The main reason for the disagreements between results obtained from XSDRNPM and albedo calculations is due to the

methodology. XSDRNPM solves the one-dimension Boltzmann transport equation using a discrete ordinates (S_n) approximation method while the albedo theory approach is an analytical and intuitive method of determining the nonleakage probabilities which also involves easily identified approximations. The four-group albedo method does lead to a very satisfactory result with an analytic and easily understandable solution. Calculations of the sensitivity of these results to variations in the relevant approximations is relegated to the next chapters which deal with gas core reactors. It is in the application to gas core reactors that the albedo method yields calculational approaches which could have major significance.

CHAPTER VI
STATIC, ONE-DIMENSIONAL NEUTRONIC ANALYSIS OF A SMALL OPTICAL-PATH-LENGTH CORE REACTOR (TWO-REGION GAS CORE REACTOR)

Introduction

Multigroup neutron albedo methods are developed in Chapters II, III, and IV, and then applied to a solid core reactor in Chapter V. The multigroup albedo technique is a generalization of the method used by Jacobs [2] to successfully predict and explain the origin of positive temperature coefficient effects in "swimming pool" research reactors, which have large optical dimensions. The ideas employed in the multigroup neutron albedo approach are actually more useful and applicable to gaseous cores than to the original "solid core" application.

A significant number of studies have been performed on many different gas core reactor concepts over the past 30 years. These studies have focused primarily on externally-moderated, gaseous core reactor designs and have generally found standard diffusion theory to be invalid in the gas core region since the gas core dimensions rarely exceed a mean free path [10].

Core and Reflector Atomic Densities

Consider a small optical-path-length core reactor surrounded by an infinite beryllium oxide reflector where it is assumed that the materials are a homogeneous mixture in the spherical core. In Table 6-1 are the core and reflector atomic densities employed.

TABLE 6-1. CORE AND REFLECTOR ATOMIC DENSITIES OF THE GAS CORE REACTOR PROBLEM

Material	Atomic Densities (atoms/(barn x cm))
Uranium - 235	.10400 E-04
Uranium - 238	.18350 E-05
Fluorine - 19	.48930 E-04
Be in BeO	.72230 E-01
Oxygen - 16	.72230 E-01

Core and Reflector Group Constants

For this problem, NITAWL [11] is used to generate a 123 fine group neutron cross-section library. The 123 fine group neutron cross-sections (30 thermal groups) are collapsed down to a 26 energy-group structure (6 thermal groups) and then to a 4 energy-group structure (one thermal group) with XSDRNPM. In Tables 6-2 and 6-3 are the core and reflector group constants, respectively.

TABLE 6-2. CORE GROUP CONSTANTS OF THE GAS CORE REACTOR PROBLEM FROM THE XSDRNPM CODE

	Four Energy-Group Structure			
	i = 1	i = 2	i = 3	i = 4
2	.19855 E+04	.86368 E+03	.44596 E+03	.94931 E+02
$\Sigma_{n,\ 2n}$	.11286 E-06	.00000	.00000	.00000
χ	.74351	.25629	.20253 E-03	.12520 E-08
Σ_a	.15107 E-04	.23846 E-04	.39035 E-03	.31475 E-02
$\nu\Sigma_f$	.36552 E-04	.43809 E-04	.53687 E-03	.64360 E-02
Σ_t	.22733 E-03	.43198 E-03	.39035 E-03	.35183 E-02
	$\Sigma_{s_{ii'}}$			
i→i′	i′ = 1	i′ = 2	i′ = 3	i′ = 4
1	.18190 E-03	.30356 E-04	.27863 E-08	.16460 E-11
2	-	.40549 E-03	.24982 E-05	.12203 E-10
3	-	-	.36193 E-03	.21137 E-05
4	-	-	-	.37083 E-03

- D in cm and Σ in cm^{-1}
- Radius of the spherical core = 70.0 cm

From Table 6-2, the ratio of core diameter to the total mean free paths in the gas core is found to be very small (~0.5 for thermal neutrons), thus, this gas core has small optical dimensions.

TABLE 6-3. REFLECTOR GROUP CONSTANTS OF THE GAS CORE REACTOR PROBLEM FROM THE XSDRNPM CODE

	Four Energy-Group Structure			
	i = 1	i = 2	i = 3	i = 4
D	.11681 E+01	.54420	.51029	.37525
$\Sigma_{n,\ 2n}$	.58851 E-02	.00000	.00000	.00000
Σ_a	.29134 E-02	.80982 E-05	.21532 E-04	.38769 E-03
Σ_t	.37032	.66685	.70088	.99156

i→i′	$\Sigma_{s_{ii'}}$		
	i′ = 2	i′ = 3	i′ = 4
1	.62765 E-01	.15147 E-04	.17131 E-11
2	-	.21315 E-01	.00000
3	-	-	.14595 E-01

- D in cm and Σ in cm^{-1}

From Table 6-3, the ratio of $\Sigma_{n,\ 2n}/(\Sigma_a + \Sigma_{n,\ 2n})$ for group 1 neutrons in the reflector is found to be very large (~0.7). This observation indicates that for the evaluation of the reflector albedos the neutron production via the Be^9 (n, 2n) Be^8 reaction must be considered.

Reflector (β) Albedos

The diffusion theory approximation within the infinite beryllium oxide reflector is valid. The neutrons produced by the Be^9 (n, 2n) 2 He^4 interaction in the reflector are accounted for in ${}_1\beta_1$, ${}_1\beta_2$, ${}_1\beta_3$, and ${}_1\beta_4$ results (see Appendix G). In Table 6-4 are the reflector albedo values obtained using the diffusion approximation (see Appendix D).

TABLE 6-4. REFLECTOR ALBEDOS OF THE GAS CORE REACTOR PROBLEM USING THE FOUR-GROUP ALBEDO THEORY

	${}_1\beta_{1'}$			
$i \rightarrow i'$	$i' = 1$	$i' = 2$	$i' = 3$	$i' = 4$
1	.28040	.30778	.11624	.18519
2	-	.62470	.15887	.13476
3	-	-	.68449	.23593
4	-	-	-	.93266

- Radius of the spherical core = 70.0 cm
- Infinite beryllium oxide reflector

From this table, the probability that a neutron from group 1 is reflected by the reflector is found to be large (${}_1\beta_1 + {}_1\beta_2 + {}_1\beta_3 + {}_1\beta_4 \cong .89$). Thus, only 11% of the group 1 neutrons impinging on the reflector are absorbed by nuclei of the BeO reflector. However, these neutron absorptions are responsible for the n, 2n gain in the system.

The Ping-Pong Decision Process

In the albedo method as applied to a gas core reactor, the description of average neutron histories is accomplished through the specification of reflector albedos and core transmittances at each neutron traversal of the core-reflector interface.

Strategy for the Gas Core Reactor Problem

Considering the fission neutron energy distribution in Table 6-2, essentially all fission neutrons are born in the first two energy groups, i.e., above 5.53 keV and below 14.92 MeV. Figure 6-1 represents the average neutron history of a group 1 neutron source in the spherical core surrounded by an infinite reflector. Because of the lengthy algebraic manipulations involved, only a sketch of the method of determining the nonleakage probabilities is presented here.

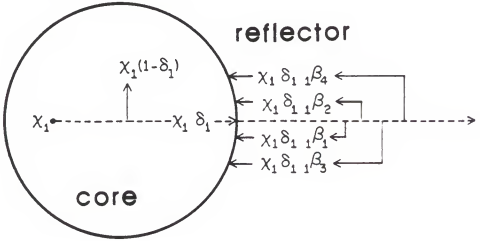


Figure 6-1. Schematic drawing showing the albedo method strategy for the gas core reactor problem.

For example, the first source of initial leakage neutrons for the evaluation of P_o and P is χ_1. The fraction of these neutrons transmitted through the core as group 1 is δ_1. The quantities returned as group 1, 2, 3, and 4 on the first core boundary traverse are $\chi_1\ \delta_1\ {}_1\beta_1$, $\chi_1\ \delta_1\ {}_1\beta_2$, $\chi_1\ \delta_1\ {}_1\beta_3$, and $\chi_1\ \delta_1\ {}_1\beta_4$, respectively. The neutron fraction, $\chi_1\ (1 - \delta_1)$, interacts in the core and can be dissected into:

(1) The fraction absorbed in the core in group 1, i.e.,

$$\chi_1 \left(1 - \delta_1\right) \frac{\Sigma_{a_1}}{\Sigma_{t_1}} ,$$

(2) The fraction scattered in the core into group 1, i.e.,

$$\chi_1 \left(1 - \delta_1\right) \frac{\Sigma_{s11}}{\Sigma_{t_1}} = I_1$$

(3) The fraction scattered in the core into group 2, i.e.,

$$\chi_1 \left(1 - \delta_1\right) \frac{\Sigma_{s12}}{\Sigma_{t_1}} = I_2$$

(4) The fraction scattered in the core into group 3, i.e.,

$$\chi_1 \left(1 - \delta_1\right) \frac{\Sigma_{s13}}{\Sigma_{t_1}} = I_3$$

(5) The fraction scattered in the core into group 4, i.e.,

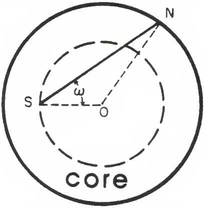
$$\chi_1 \left(1 - \delta_1\right) \frac{\Sigma_{s14}}{\Sigma_{t_1}} = I_4 .$$

Thus, in this first run, there are created four core secondary sources, whose strengths are I_1, I_2, I_3, and I_4, and four reflector secondary sources. $\chi_1 \left(1 - \delta_1\right) \frac{\Sigma_{a_1}}{\Sigma_{t_1}}$ is the quantity absorbed in the core as group 1.

For the next steps, it is required to consider the presence of all secondary sources in order to estimate the total quantities that will be absorbed as groups 1, 2, 3, and 4 in the core. To help understand the evaluation of P_o and P, the contribution of one group 4 neutron source is detailed in Appendix F.

Transmission Coefficients for the Core and Reflector Secondary Sources

The core transmission coefficients for the reflector secondary sources for each energy group, γ_i, are evaluated as described in Chapter III. By calculating the core transmission coefficients for the core secondary sources for each energy group, δ_i, it is assumed that: (1) the neutron sources are isotropic and (2) the neutrons are homogeneously distributed at the surface of a sphere of radius R_o given by $R_o = \left[\frac{R_c^3}{2} \right]^{1/3}$, i.e., this immaginary sphere has half volume of the core. With the aid of Figure 6-2, the evaluation of δ_i values can be visualized.



$R_c = \overline{ON}$,

$R_o = \overline{OS}$, and

$\overline{SN} = R_o \cos \omega + \sqrt{R_c^2 - R_o^2 \sin^2 \omega}$

Figure 6-2. Schematic drawing for evaluation of the core transmission coefficients for the core secondary sources.

The δ_i values are given by

$$\delta_i = \frac{\int_{\omega=0}^{\pi} \exp\left[-\Sigma_{t_i} \times \overline{SN} \right] \frac{\sin \omega}{2} \, d\,\omega}{\int_{\omega=0}^{\pi} \frac{\sin \omega}{2} \, d\,\omega} \tag{6-1}$$

where

$$\overline{SN} = R_o \cos \omega + \sqrt{R_c^2 - R_o^2 \sin^2 \omega} \quad \text{and}$$

$$R_o = \frac{R_c}{\sqrt[3]{2}} \quad .$$

In Table 6-5 are the transmission coefficients for all secondary sources.

TABLE 6-5. TRANSMISSION COEFFICIENTS FOR THE CORE (δ_i) AND REFLECTOR (γ_i) SECONDARY SOURCES OF THE GAS CORE REACTOR PROBLEM

	Four-Group Albedo Theory Method			
	i = 1	i = 2	i = 3	i = 4
γ_i	.98425	.97036	.94900	.78963
δ_i	.98811	.97759	.96134	.83643

- Radius of the spherical core = R_c = 70 cm.
- $R_o = R_c / \sqrt[3]{2}$

Dissection of P_o and P

As was done for the solid core reactor problem, it is also interesting to evaluate: (1) the fraction of neutrons which, having been produced by fission in the core, never traverse the core boundary, but are absorbed as group i, here denoted by A_{o_i}, and (2) the fraction of neutrons which are produced by fission in the core, spend time in the reflector but are eventually absorbed by nuclei of the core as group i, here denoted by A_i. The program ALB2 used for these numerical calculations is in Appendix G. In Table 6-6 are the probabilities, A_{o_i} and A_i, where

$$P_o = \sum_{i=1}^{4} A_{o_i} \quad \text{and} \quad P = \sum_{i=1}^{4} A_i \,.$$

TABLE 6-6. NONLEAKAGE PROBABILITIES, A_{o_i} AND A_i, OF THE GAS CORE REACTOR PROBLEM USING THE FOUR-GROUP ALBEDO THEORY METHOD

	Four-Group Albedo Theory Method			
	i = 1	i = 2	i = 3	i = 4
A_{o_i}	.59772 E-03	.32561 E-03	.69758 E-06	.59687 E-09
A_i	.30723 E-03	.26766 E-02	.29157 E-01	.50246

- Radius of the spherical core = R_c = 70 cm.
- $R_o = R_c / \sqrt[3]{2}$

From this table, the total principal nonleakage probability, P_o, is found to be very small, .92404 E-03, while the total secondary nonleakage probability, P, is now very high, i.e., .53553. Thus, it is very clear that P almost represents 100% of the total nonleakage probability value for this gas core reactor problem.

Effective Neutron Multiplication Factor, k_{eff}

The neutrons produced by the (n, 2n) interaction in the gas core are accounted for in k_{eff} evaluation. Thus, the k_{eff} value is given by

$$k_{eff} = \sum_{i=1}^{4} \left(A_{o_i} + A_i\right) \frac{\nu_i \Sigma_{f_i} + 2 \Sigma_{n,\,2n_i}}{\Sigma_{a_i}} \quad \text{and}$$

$$k_{eff} = 1.0752.$$

Comparison: XSDRNPM Versus Albedo Method Results

One-dimensional static neutronic analysis has also been performed on the gas core reactor problem with XSDRNPM, four-group, S_4 P_3 , computer code. It is desirable to compare the results obtained from this code because such comparisons aid in determining the accuracy and reliability of the obtained results. In Table 6-7 are the total nonleakage probability, P_{TNL}, and effective neutron multiplication factor, k_{eff}, for the system as obtained from XSDRNPM and the four-group albedo method.

TABLE 6-7. TOTAL NONLEAKAGE PROBABILITY, P_{TNL}, AND EFFECTIVE NEUTRON MULTIPLICATION FACTOR, k_{eff}, VALUES OF THE GAS CORE REACTOR PROBLEM USING THE XSDRNPM CODE AND THE ALBEDO METHOD

	XSDRNPM	Albedo
P_o	-	.92404 E-03
P	-	.53460
$P_{TNL} = P_o + P$	.56869	.53553
k_{eff}	1.1374	1.0752
CPU Time	1.0 sec	< 1 sec

- Radius of the spherical core = R_c = 70 cm
- $R_o = R_c / \sqrt[3]{2}$

- Infinite reflector for the albedo results
- Reflector 100 cm thick for XSDRNPM values
- XSDRNPM parameters: four group, S_4 P_3, number of spacial intervals = 60, overall convergence = 1.0 E-04
- XSDRNPM calculation on an IBM 3090-600J
- Four-group albedo method values on a 386-SX personal computer

Table 6-7 shows that, for the examined configuration, the albedo theory approximation is found to underpredict k_{eff} by about 5.5% relative to the S_4 P_3 result.

This rather large disagreement suggests that some of the approximations used in the albedo method deserve further attention. Sensitivity of results to some of these approximations are considered in the next section.

Analyses of Sensitivity

Comparative neutronic calculations are performed on the gas core reactor problem in one dimension to determine the impact of the variation of some parameters which are either

chosen intuitively in the method, or are of unconventional interest. The cases chosen are: (1) R_o variation, (2) assume $P_o = 0.0$, (3) (n, 2n) reaction neglect in the reflector, (4) assume anisotropic secondary source for core transmittances, (5) assume anisotropic source for reflector albedos, and (6) combination of (4) and (5).

R_o Variation

It has been assumed that the core neutrons are uniformly produced on the surface of a sphere of radius R_o. R_o can range from zero to the core radius. In Table 6-8 are the nonleakage probability and effective neutron multiplication factor values for different R_o.

TABLE 6-8. NONLEAKAGE PROBABILITY AND EFFECTIVE NEUTRON MULTIPLICATION FACTOR VALUES OF THE GAS CORE REACTOR PROBLEM BY VARYING R_o

	Four-Group Albedo Theory Method			
R_o	P_o	P	$P_{TNL} = P_o + P$	k_{eff}
0.0	.12353 E-02	.53539	.53663	1.0774
R_o^*	.92404 E-03	.53460	.53553	1.0752
R_c	.60982 E-03	.53376	.53437	1.0730

- Radius of the spherical core = R_c = 70 cm.
- $R_o^* = R_c / \sqrt[3]{2}$ = .55559 E+02 cm

Table 6-8 shows that less than 0.4% difference exists among k_{eff} values due to varying R_o and the highest value of k_{eff} is for R_o = 0.0. This is expected since the variation of R_o yields effects on core transmittance which completely explain these results.

P_o Neglect

From previous results it is appropriate to use the approximation that all fission neutrons escape from the gas core without core interaction. In Table 6-9 are the total nonleakage probability, P_{TNL}, and effective neutron multiplication factor, k_{eff}, using P_o = 0.

TABLE 6-9. TOTAL NONLEAKAGE PROBABILITY, P_{TNL}, AND EFFECTIVE NEUTRON MULTIPLICATION FACTOR, k_{eff}, VALUES OF THE GAS CORE REACTOR PROBLEM NEGLECTING P_o AND VARYING R_o

	Four-Group Albedo Theory Method	
R_o	$P_{TNL} = P_o + P,\ P_o = 0.0$	k_{eff}
0.0	.53653	1.0766
R_o^*	.53536	1.0747
R_c	.53433	1.0727

- Radius of the spherical core = R_c = 70 cm.
- $R_o^* = R_c / \sqrt[3]{2}$ = .55559 E+02 cm

Tables 6-8 and 6-9 show that the P_o neglect gives smaller k_{eff} values but less than 0.07% difference exists among k_{eff} values considering the same imaginary R_o. This is expected

since P_o = 0.0 yields an increase in the fraction of neutrons that leaks from the core for the first time.

Be^9(n, 2n) Be^8 Reaction Neglect

The neutrons produced by the Be^9 (n, 2n) 2 He^4 interaction in the reflector are accounted for in ${}_1\beta_1$, ${}_1\beta_2$, ${}_1\beta_3$, and ${}_1\beta_4$ albedos, and result in k_{eff} variation. Because the ratio $\Sigma_{n,\,2n}/\Sigma_{t_1}$ (~1.6%) is small, it might seem appropriate to neglect the effect of the reaction. In Table 6-10 is compared the group 1 albedos and k_{eff} including the (n, 2n) reaction and neglecting it.

TABLE 6-10. GROUP 1 REFLECTOR ALBEDOS AND EFFECTIVE NEUTRON MULTIPLICATION FACTOR OF THE GAS CORE REACTOR PROBLEM FOR EITHER $\Sigma_{n,\,2n} = 0$ OR $\Sigma_{n,\,2n} \neq 0$

$\Sigma_{n,\,2n}$	Four-Group Albedo Theory Method				
	${}_1\beta_{i'}$				k_{eff}
	i' = 1	i' = 2	i' = 3	i' = 4	
= 0.0	.25992	.29529	.10932	.17031	1.0076
≠ 0.0	.28040	.30778	.11624	.18519	1.0752

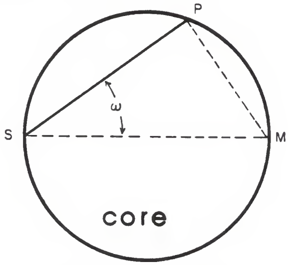
- Radius of the spherical core = R_c = 70.0 cm.
- $R_o = R_c / \sqrt[3]{2}$

From Table 6-10, it is clear that neglect of the (n, 2n) interaction in the reflector yields for the probability that a neutron from group 1 be reflected by the reflector a value of .83, i.e., ${}_1\beta_1 + {}_1\beta_2 + {}_1\beta_3 + {}_1\beta_4 \cong .83$. Including the reaction yields a value of nearly .89. Finally, the Be^9 (n, 2n) Be^8 interaction neglect leads an error of about 6.3%

in k_{eff} value for the examined configuration. The (n, 2n) reaction is certainly significant.

<u>Anisotropic Secondary Source on the Reflector-Core Interface for Core Transmittance Calculation</u>

Previously, the core transmission coefficients, γ_i, were evaluated by considering an isotropic neutron source on a spherical core of diameter 2R. The angular neutron flux distribution deep inside a homogeneous medium is nearly isotropic because leakage processes, which are anisotropic by nature, are relatively unimportant and neutron production processes, such as fission, tend to be isotropic. However, near strong discontinuities in material properties, such as the core-reflector interface, the angular neutron flux distribution is usually relatively anisotropic. Consider a linear anisotropic neutron source emitting S_R neutrons per second (see Figure 6-3).



$S_R = S_o\ (1 + A\ \mu)$ where

$\mu = \cos\omega\ ,\ 0 \le \mu\ 1$.

and the parameter A represents an angular coefficient.

$\overline{SP} = 2R\cos\omega$
$\overline{SM} = 2R$

Figure 6-3. Schematic drawing for evaluation of the core transmission coefficients, γ_i, due to a linear anisotropic neutron source on a spherical core.

The core transmission coefficients, γ_i, for this linear anisotropic neutron source are given by

$$\gamma_i = \frac{\int_{\omega=0}^{\pi/2} \exp\left[-\Sigma_{t_i} \times 2R \times \cos\omega\right](1 + A\cos\omega)\, 2\pi \sin\omega\, d\omega}{\int_{\omega=0}^{\pi/2} (1 + A\cos\omega)\, 2\pi\sin\omega\, d\omega} \tag{6-2}$$

In Table 6-11 are compared the core transmission coefficients, γ_i, and effective neutron multiplication factor for different values of the angular anisotropy coefficient, A.

TABLE 6-11. CORE TRANSMISSION COEFFICIENTS, γ_i, AND EFFECTIVE NEUTRON MULTIPLICATION FACTOR OF THE GAS CORE REACTOR PROBLEM BY VARYING THE ANGULAR ANISOTROPY COEFFICIENT, A.

A	Four-Group Albedo Theory Method				
	γ_i				
	i = 1	i = 2	i = 3	i = 4	k_{eff}
0.0	.98424	.97037	.94900	.78963	1.0752
0.5	.98321	.96841	.94566	.77671	1.0922
1.0	.98251	.96710	.94344	.76811	1.1028

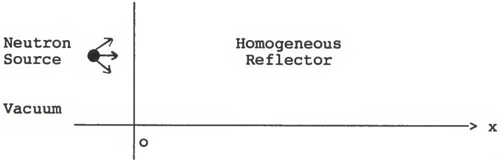
Table 6-11 shows that when the angular anisotropy coefficient, A, increases, the core transmission coefficients, γ_i, decrease, which clearly yields an increase in the effective neutron multiplication factor.

<u>Anisotropic Source on the Reflector-Core Interface for Reflector Albedo Calculation</u>

Directional dependence of the neutron flux is greatest close to a boundary. To this point, the reflector albedo values were evaluated by using the diffusion approximation (P_1) to the Boltzmann transport equation. Consider the reflector albedo evaluation employing the next relevant higher-order approximation (P_3) to the Boltzmann transport equation. To simplify the calculation, the following relationship among the P_1 and P_3 reflector albedo values is assumed:

$$\beta_{P_3}^{\text{spherical}} = \beta_{P_1}^{\text{spherical}} \times \frac{\beta_{P_3}^{\text{plane symmetry}}\ (\text{for } B \neq 0)}{\beta_{P_3}^{\text{plane symmetry}}\ (\text{for } B = 0)} . \tag{6-3}$$

Consider the problem of a linear anisotropic neutron source emitting S_o neutrons per second that illuminates a plane surface of a homogeneous half-space as shown in Figure 6-4.



$S(\mu) = \frac{S_o}{2\pi}(1 + B\mu)$ where the parameter B represents an angular coefficient.

Figure 6-4. Schematic drawing for evaluation of the reflector albedo values due to a linear anisotropic neutron source in P_3 plane-symmetry geometry.

Plane-symmetry, P_3, four-group transport theory reflector albedo evaluation

Because of the lengthy algebraic manipulations involved, only an outline of the method of determining ${}_1\beta_4$ value is presented here. Results for all the P_3 albedos are presented in Appendix G (see program ALB3). The angular neutron flux expression for group i is

$$\phi_i (x, \mu) = \sum_{n=0}^{3} \frac{2n + 1}{4\pi} \phi_{i,n} (x) P_n (\mu) \tag{6-4}$$

which yields a coupled set of sixteen linear, first-order ordinary differential equations:

(1) For i = 1

$$\begin{aligned}
&\frac{d}{dx} \phi_{11} + \Sigma_{R_1} \phi_{10} = 0 \\
&\frac{d}{dx} \phi_{10} + 2 \frac{d}{dx} \phi_{12} + 3 \Sigma_{t_1} \phi_{11} = 0 \\
&2\frac{d}{dx} \phi_{11} + 3 \frac{d}{dx} \phi_{13} + 5 \Sigma_{t_1} \phi_{12} = 0 \\
&3\frac{d}{dx} \phi_{12} + 7 \Sigma_{t_1} \phi_{13} = 0
\end{aligned} \tag{6-5}$$

(2) For i = 2

$$\begin{aligned}
&\frac{d}{dx} \phi_{21} + \Sigma_{R_2} \phi_{20} = \Sigma_{s12} \phi_{10} \\
&\frac{d}{dx} \phi_{20} + 2 \frac{d}{dx} \phi_{22} + 3 \Sigma_{t_2} \phi_{21} = 0 \\
&2\frac{d}{dx} \phi_{21} + 3 \frac{d}{dx} \phi_{23} + 5 \Sigma_{t_2} \phi_{22} = 0 \\
&3\frac{d}{dx} \phi_{22} + 7 \Sigma_{t_2} \phi_{23} = 0
\end{aligned} \tag{6-6}$$

(3) For i = 3

$$\frac{d}{dx}\phi_{31} + \Sigma_{R_3}\phi_{30} = \Sigma_{s_{13}}\phi_{10} + \Sigma_{s_{23}}\phi_{20}$$

$$\frac{d}{dx}\phi_{30} + 2\frac{d}{dx}\phi_{32} + 3\Sigma_{t_3}\phi_{31} = 0$$

$$2\frac{d}{dx}\phi_{31} + 3\frac{d}{dx}\phi_{33} + 5\Sigma_{t_3}\phi_{32} = 0 \qquad (6\text{-}7)$$

$$3\frac{d}{dx}\phi_{32} + 7\Sigma_{t_3}\phi_{33} = 0$$

(4) For i = 4

$$\frac{d}{dx}\phi_{41} + \Sigma_{R_4}\phi_{40} = \Sigma_{s_{14}}\phi_{10} + \Sigma_{s_{24}}\phi_{20} + \Sigma_{s_{34}}\phi_{30}$$

$$\frac{d}{dx}\phi_{40} + 2\frac{d}{dx}\phi_{42} + 3\Sigma_{t_4}\phi_{41} = 0$$

$$2\frac{d}{dx}\phi_{41} + 3\frac{d}{dx}\phi_{43} + 5\Sigma_{t_4}\phi_{42} = 0 \qquad (6\text{-}8)$$

$$3\frac{d}{dx}\phi_{42} + 7\Sigma_{t_4}\phi_{43} = 0$$

The expression for ${}_1\beta_4$ is

$${}_1\beta_4 = \frac{2\pi\int_{\mu=0}^{-1}\mu\,\phi_4\,(x=0,\,\mu)\,d\mu}{2\pi\int_{\mu=0}^{+1}\frac{S_o}{2\pi}\,(1+B\,\mu)\,d\mu} \,. \qquad (6\text{-}9)$$

All boundary conditions and albedo expressions are presented in program ALB3 (see Appendix G).

In Table 6-12 are compared the total nonleakage probability and effective neutron multiplication factor for different values of the angular coefficient, B.

TABLE 6-12. TOTAL NONLEAKAGE PROBABILITY, P_{TNL}, AND EFFECTIVE NEUTRON MULTIPLICATION FACTOR OF THE GAS CORE REACTOR PROBLEM BY VARYING THE ANGULAR COEFFICIENT, B

B	Four-Group Albedo Theory Method			
	P_o	P	P_{TNL}	k_{eff}
0.0	.92404 E-03	.53460	.53553	1.0752
0.5	.92404 E-03	.53548	.53640	1.0773
1.0	.92404 E-03	.53601	.53694	1.0786

- Radius of the spherical core = R_c = 70.0 cm
- $R_o = R_c / \sqrt[3]{2}$

Table 6-12 shows that when the angular coefficient, B, increases, the effective neutron multiplication factor of the examined configuration increases. The P_3 reflector albedo values (in plane symmetry) are presented in Table 6-13. This table shows that when the angular coefficient, B, increases, ${}_1\beta_1$, ${}_2\beta_2$, ${}_3\beta_3$, ${}_4\beta_4$, and ${}_1\beta_2$ decrease, but ${}_1\beta_3$, ${}_1\beta_4$, ${}_2\beta_3$, ${}_2\beta_4$, and ${}_3\beta_4$ increase (outweighing these contributions for the k_{eff} value).

TABLE 6-13. P_3 REFLECTOR ALBEDO VALUES IN PLANE-SYMMETRY GEOMETRY BY VARYING THE ANGULAR ANISOTROPY COEFFICIENT, B.

$_i\beta_{i'}$	B		
	0.0	0.5	1.0
$_2\beta_2$	.40604	.40243	.40002
$_1\beta_2$	.30134	.29953	.29832
$_1\beta_3$	.10219	.10323	.10393
$_1\beta_4$	.14748	.15038	.15230
$_2\beta_2$	.68207	.67794	.67519
$_1\beta_3$	.15038	.15[illegible]30	.15189
$_2\beta_4$	.12007	.12232	.12381
$_3\beta_3$	.73584	.73205	.72952
$_3\beta_4$	.21694	.21973	.22160
$_4\beta_4$	.95934	.95852	.95797

Anisotropic Neutron Source on the Reflector-Core Interface for Reflector Albedo and Core Transmittance Evaluations

The anisotropy of the neutron flux is greatest close to the reflector-core interface of the gas core reactor problem examined. In Table 6-14 is summarized the results of calculations of k_{eff} by XSDRNPM and the four-group albedo approach including core and reflector anisotropy factors $A = B = 1.5$.

TABLE 6-14. COMPARISON OF k_{eff} RESULTS OF THE GAS CORE REACTOR PROBLEM USING VARIOUS CORE DENSITY FACTOR (D.F.) FROM THE XSDRNPM CODE AND THE FOUR-GROUP ALBEDO METHOD WITH ANISOTROPY FACTORS A = B = 1.5

D.F.	P_{TNL}		k_{eff}		k_{eff} Relative Error
	XSDRNPM	Albedo	XSDRNPM	Albedo	
0.5	.47460	.47088	.95712	.95154	.583 E-02
1.0	.56869	.56469	1.1374	1.1329	.396 E-02
2.0	.63460	.63470	1.2511	1.2576	.520 E-02

- Radius of the spherical core = R_c = 70.0 cm
- $R_o = R_c / \sqrt[3]{2}$
- Angular coefficients: A = 1.5 and B = 1.5
- XSDRNPM parameters: Four-group, S_4 P_3, number of spacial intervals = 60, overall convergence = 1.0 E-04.
- k_{eff} relative error = $\dfrac{k_{eff}\,(\text{XSDRNPM}) - k_{eff}(\text{albedo})}{k_{eff}\,(\text{XSDRNPM})}$

From Table 6-14, it is clear that, if A = B = 1.5 is assumed, the k_{eff} values for density factors in the range 0.5 to 2.0 as calculated by the albedo method are in close agreement with those calculated by XSDRNPM. In contrast, the results presented in Table 6-7 for the case D.F. = 1.0 and A = B = 0.0 (i.e., for isotropic-incident albedos) indicate significantly less correspondence. It is concluded that the anisotropic-incident effects are crucial for k_{eff} calculations. However, this effect should be far less important in reactivity (i.e., change in k_{eff} due to parameter variation) determination.

It should be emphasized, again, that the ability to recognize the physical phenomena origin of k_{eff} variation through albedo parameter dependence is the most interesting aspect of the multigroup albedo approach. In the next chapter, another aspect of the value of these methods is addressed, viz., the resulting ability to approximate effects on k_{eff} of symmetry perturbations which could not be included in conventional approaches (e.g., XSDRNPM application) without incurring considerable computational time and expense.

CHAPTER VII
STATIC, ONE-DIMENSIONAL NEUTRONIC ANALYSIS OF A SMALL OPTICAL-PATH-LENGTH CORE REACTOR (THREE-REGION GAS CORE REACTOR)

Introduction

As stated in Chapter VI the ideas employed in the multigroup neutron albedo approach are more useful and applicable to gaseous core reactors than to the "solid core" applications. Figure 7-1 illustrates a gaseous core (fuel radius R_c), intermediate gaseous region (thickness $R_p - R_c$), and solid moderator (thickness $R_m - R_p$) of a reactor system similar to a proposed concept of a gas core propulsion reactor [12]. For such a system, crucial considerations of nuclear reactor system dynamics are not approachable (at reasonable cost) with conventional calculational procedures.

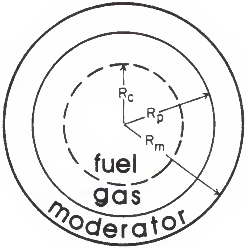


Figure 7-1. Schematic drawing of a three-region gas core reactor.

Three-Region Gas Core Reactor Atomic Densities

Consider a spherical cavity region consisting of a ball of gaseous fuel suspended in an intermediate gas (hydrogen). The cavity is surrounded by an infinite beryllium oxide reflector-moderator. In Table 7-1 are the plausible atomic densities for such a three-region gas core reactor problem.

TABLE 7-1. ATOMIC DENSITIES OF THE THREE-REGION GAS CORE REACTOR PROBLEM

Region	Material	Atomic Density (atoms/(barn x cm))
Fuel	Uranium - 238	.10400 E-04
	Uranium - 238	.18350 E-05
	Fluorine - 19	.49830 E-04
Intermediate Gas	Hydrogen - 1	.49830 E-04
Moderator Reflector	Be in BeO	.7223 E-01
	Oxygen - 16	.7223 E-01

Three-Region Gas Core Reactor Group Constants

The fuel and reflector group constants are obtained from XSDRNPM calculations (see Tables 6-2 and 6-3, respectively). The fast group constants for hydrogen are obtained from PHROG [13] calculations while the thermal group constants are obtained from BRT [14] calculations by using the free gas kernel option. The molecular binding of the hydrogen molecule is ignored (i.e., atomic hydrogen is assumed).

In Table 7-2 are the hydrogen group constants.

TABLE 7-2. HYDROGEN GROUP CONSTANTS OF THE THREE-REGION GAS CORE REACTOR PROBLEM

	Four Energy-Group Structure			
	i = 1	i = 2	i = 1	i = 4
Σ_a	.00000	.00000	.36286 E-06	.69789 E-05
Σ_{tr}	.76168 E-04	.12559 E-03	.30068 E-03	.47675 E-03
Σ_t	.13829 E-03	.54005 E-03	.98166 E-03	.12778 E-02
	$\Sigma_{s_{ii'}}$			
i→i′	i′ = 1	i′ = 2	i′ = 3	i′ = 4
4	.67670 E-04	.70140 E-04	.47548 E-06	.15990 E-09
2	-	.43390 E-03	.10612 E-03	.35687 E-07
3	-	-	.85897 E-03	.12234 E-03
4	-	-	-	.12709 E-02

- Σ in cm^{-1}
- Fuel radius = R_c = 100.0 cm.
- Hydrogen thickness = $R_p - R_c$ = 50.0 cm.

From Table 7-2, the ratio of hydrogen thickness to the total mean free paths in the hydrogen region is found to be very small (~0.06 for thermal neutrons). Thus, this hydrogen region is also of small optical dimensions.

Reflector (β) Albedos

The diffusion approximation within the infinite beryllium oxide reflector is employed. The neutrons produced by the Be^9 (n, 2n) Be^8 interaction are accounted for in ${}_1\beta_1$, ${}_1\beta_2$, ${}_1\beta_3$, and ${}_1\beta_4$ values. In Table 7-3 are the reflector albedo values obtained.

TABLE 7-3. REFLECTOR ALBEDOS OF THE THREE-REGION GAS CORE REACTOR PROBLEM USING THE FOUR-GROUP ALBEDO THEORY

$i \to i'$	${}_i\beta_{i'}$			
	$i' = i$	$i' = 2$	$i' = 3$	$i' = 4$
1	.29516	.31344	.11856	.18865
2	-	.63572	.16100	.13658
3	-	-	.69559	.23881
4	-	-	-	.94340

- Internal radius of the spherical reflector = R_m = 150.0 cm.
- Infinite beryllium oxide reflector.

From Table 7-3, the probability that a neutron from group 1 be reflected by the reflector is found to be very large (${}_1\beta_1 + {}_1\beta_2 + {}_1\beta_3 + {}_1\beta_4 \cong .92$). Thus, 8% of the incident neutrons from group 1 are absorbed by the reflector.

The Ping-Pong Decision Process

Assumptions established to facilitate the neutronic analyses are as follows: (1) the fuel is uniformly distributed within the central portion of the cavity volume and (2) no mixing is assumed to occur between fuel and hydrogen in the cavity. In the albedo method applied to the three-region gas core reactor problem, the description of average neutron histories is accomplished through the specification of reflector reflection and fuel and hydrogen transmission probabilities at each neutron traversal of the fuel-hydrogen-reflector interfaces.

Strategy for the Three-Region Gas Core Reactor Problem

As previously presumed, χ_3 and χ_4 are considered neglígle, i.e., all fission neutrons are emitted in groups 1 and 2. Figure 7-2 represents the average neutron history of a group 1 neutron source in the three-region gas core reactor problem. Because of the lengthy algebraic manipulations involved, only a sketch of the method of determining the nonleakage probabilities for the fuel and hydrogen regions is presented here.

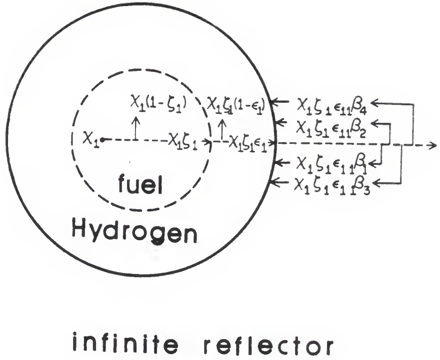


Figure 7-2. Schematic drawing showing the albedo method strategy for the three-region gas core reactor problem.

With reference to Figure 7-2, the first source of initial leakage neutrons for the evaluation of P_o and P is χ_1. The fraction of these neutrons transmitted through the fuel region as group 1 is ζ_1. After that the fraction of these neutrons transmitted through the hydrogen region as group 1 is ϵ_1. The quantities returned as group 1, 2, 3, and 4 on the first cavity boundary traverse are $\chi_1\ \zeta_1\ \epsilon_1\ {}_1\beta_1$, $\chi_1\ \zeta_1\ \epsilon_1\ {}_1\beta_2$, $\chi_1\ \zeta_1\ \epsilon_1\ {}_1\beta_3$, and $\chi_1\ \zeta_1\ \epsilon_1\ {}_1\beta_4$, respectively. The neutron fraction, $\chi_1(1 - \zeta_1)$ interacts in the fuel region. It can be dissected into the contributions:

(1) The fraction absorbed in the fuel as group 1, i.e.,

$$\chi_1(1 - \zeta_1)(\Sigma^F_{a_1} / \Sigma^F_{t_1}) \ ,$$

(2) The fraction scattered in the fuel as group 1, i.e.,

$$\chi_1(1 - \zeta_1)(\Sigma^F_{s_{11}} / \Sigma^F_{t_1}) = F_1 \ ,$$

(3) The fraction scattered in the fuel as group 2, i.e.,

$$\chi_1(1 - \zeta_1)(\Sigma^F_{s_{12}} / \Sigma^F_{t_1}) = F_2 \ ,$$

(4) The fraction scattered in the fuel as group 3, i.e.,

$$\chi_1(1 - \zeta_1)(\Sigma^F_{s_{13}} / \Sigma^F_{t_1}) = F_3 \ , \text{ and}$$

(5) The fraction scattered in the fuel as group 4, i.e.,

$$\chi_1(1 - \zeta_1)(\Sigma^F_{s_{14}} / \Sigma^F_{t_1}) = F_4 \ .$$

The neutron fraction $\chi_1\ \zeta_1\ (1 - \epsilon_1)$ interacts in the hydrogen region. It can be dissected into the contributions:

(1) The fraction absorbed by hydrogen as group 1, i.e.,

$$\chi_1\ \zeta_1\ (1 - \epsilon_1)(\Sigma^H_{a_1} / \Sigma^H_{t_1}) ,$$

(2) The fraction scattered in the hydrogen region as group 1, i.e.,

$$\chi_1\ \zeta_1\ (1 - \epsilon_1)(\Sigma^H_{s_{11}} / \Sigma^H_{t_1}) = E_1 ,$$

(3) The fraction scattered in the hydrogen region as group 2, i.e.,

$$\chi_1\ \zeta_1\ (1 - \epsilon_1)(\Sigma^H_{s_{12}} / \Sigma^H_{t_1}) = E_2 ,$$

(4) The fraction scattered in the hydrogen region as group 3, i.e.,

$$\chi_1\ \zeta_1\ (1 - \epsilon_1)(\Sigma^H_{s_{13}} / \Sigma^H_{t_1}) = E_3 ,\ \text{and}$$

(5) The fraction scattered in the hydrogen region as group 4, i.e.,

$$\chi_1\ \zeta_1\ (1 - \epsilon_1)(\Sigma^H_{s_{14}} / \Sigma^H_{t_1}) = E_4 .$$

Thus, in the first cavity traversal, there are created four fuel secondary neutron sources whose strengths are F_1, F_2, F_3, and F_4, four hydrogen secondary neutron sources whose strengths are E_1, E_2, E_3, and E_4, and four reflector secondary neutron sources. Moreover, $\chi_1(1 - \zeta_1)(\Sigma^F_{a_1} / \Sigma^F_{t_1})$ is the fraction absorbed in the fuel region as group 1 while $\chi_1\ \zeta_1(1 - \epsilon_1)(\Sigma^H_{a_1} / \Sigma^H_{t_1})$ is the fraction absorbed by hydrogen nuclei as group 1.

For the next steps, it is required to consider the presence of all secondary sources (i.e., 12 sources) in

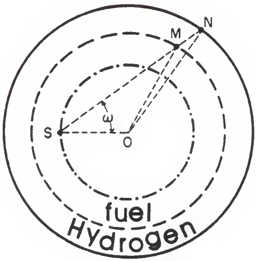
order to estimate the total fractions that are absorbed as group 1, 2, 3, and 4 in the fuel and hydrogen regions.

Transmission Coefficients for the Fuel, Hydrogen, and Reflector Secondary Neutron Sources

The fuel-hydrogen-reflector neutron transfer is described by means of three point-of-view neutron source transmission coefficients and are listed below.

First point of view: Fuel secondary neutron sources

The fuel secondary neutron sources can "see" the fuel region and the hydrogen region. Two transmission coefficients are of interest and have been identified as ζ_1 and ϵ_1. With the aid of Figure 7-3 these transmission coefficient evaluations can be better understood.



$\overline{OS} = R_o$; $\overline{OM} = R_c$

$\overline{ON} = R_p$

Thus,

$$\overline{SM} = R_o \cos\omega + \sqrt{R_c^2 - R_o^2 \sin^2 \omega}$$

$$\overline{SN} = R_o \cos\omega + \sqrt{R_p^2 - R_o^2 \sin^2 \omega}$$

$$\overline{MN} = \sqrt{R_p^2 - R_o^2 \sin^2 \omega} - \sqrt{R_c^2 - R_o^2 \sin^2 \omega}$$

Figure 7-3. Schematic drawing for the transmission coefficient evaluation under fuel secondary neutron source point of view for the three-region gas core reactor problem.

In calculating the transmission coefficients for each energy group it is assumed that: (1) the neutron sources are isotropic and (2) the neutrons are homogeneously distributed at the surface of a sphere of radius R_o given by $R_o = [\frac{R_c^3}{2}]^{1/3}$, i.e., this imaginary sphere has half volume of the fuel region volume. The ζ_i are given by

$$\zeta_i = \frac{\int_{\omega=0}^{\pi} \exp\left[-\Sigma_{ti}^{F} \text{ x } \overline{SM}\right] \frac{\sin \omega}{2} \, d\,\omega}{\int_{\omega=0}^{\pi} \frac{\sin \omega}{2} \, d\,\omega} \tag{7-1}$$

where

$$\overline{SM} = R_o \cos\omega + \sqrt{R_c^2 - R_o^2 \sin^2 \omega} \quad \text{and} \quad R_o = R_o / \sqrt[3]{2} \; .$$

The ϵ_i are given by

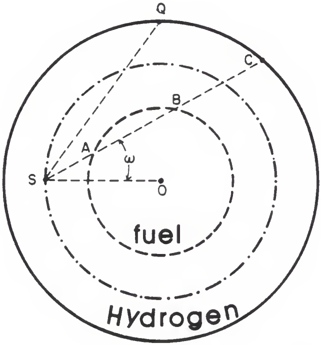
$$\epsilon_i = \frac{\int_{\omega=0}^{\pi} \exp[- \Sigma_{ti}^{H} \text{ x } \overline{MN}] \frac{\sin \omega}{2} \, d\,\omega}{\int_{\omega=0}^{\pi} \frac{\sin \omega}{2} \, d\,\omega} \tag{7-2}$$

where

$$\overline{MN} = \sqrt{R_p^2 - R_o^2 \sin^2 \omega} - \sqrt{R_c^2 - R_o^2 \sin^2 \omega}$$

<u>Second point of view: Hydrogen secondary neutron sources</u>

For the hydrogen neutron sources there are four types of relevant transmission coefficients. They involve: (1) when a neutron crosses the hydrogen-fuel-hydrogen region, and (2) when a neutron crosses only the hydrogen region. With the aid of Figure 7-4, these transmission coefficient evaluations can be visualized and derived.



$\overline{OB} = R_c$; $\overline{OC} = R_p$

$\overline{OS} = R_H$

Thus,

$$\overline{SQ} = R_H \cos\omega + \sqrt{R_p^2 - R_H^2 \sin^2\omega}$$

$$\overline{SA} = R_H \cos\omega - \sqrt{R_c^2 - R_H^2 \sin^2\omega}$$

$$\overline{AB} = 2\sqrt{R_c^2 - R_H^2 \sin^2\omega} \quad \text{and}$$

$$\overline{BC} = \sqrt{R_p^2 - R_H^2 \sin^2\omega} - \sqrt{R_c^2 - R_H^2 \sin^2\omega}$$

For $0 \le \omega \le \pi$

Figure 7-4. Schematic drawing for the transmission coefficient evaluation under hydrogen secondary neutron source point of view for the three-region gas core reactor problem.

In calculating the transmission coefficients for each energy group it is assumed that: (1) the neutron sources are isotropic and (2) the neutrons are homogeneously distributed at the surface of a sphere of radius R_H given by

$R_H = \left[\frac{R_p^3 + R_c^3}{2}\right]^{1/3}$, i.e., this imaginary sphere has the average volume of the fuel region and cavity region volumes. The relevant transmission coefficient values are given by

(1) $\overline{SA}$ hydrogen path

$$\xi_{SA,i} = \frac{\int_{\omega=0}^{\omega_0} \exp\left[-\Sigma_{t_i}^H \times \overline{SA}\right] \frac{\sin\omega}{2}\, d\omega}{\int_{\omega=0}^{\omega_0} \frac{\sin\omega}{2}\, d\omega} \tag{7-3}$$

where

$$\overline{SA} = R_H \cos\omega - \sqrt{R_c^2 - R_H^2 \sin\omega}\ , \quad R_H = \left[\frac{R_p^3 + R_c^3}{2}\right]^{1/3} ,$$

$$\text{and } \omega_o = \sin^{-1}\left[\frac{R_c}{R_H}\right]$$

(2) $\overline{AB}$ fuel path

$$\xi_{AB,i} = \frac{\int_{\omega=0}^{\omega_0} \exp\left[-\Sigma_{t_i}^F \times \overline{AB}\right] \frac{\sin\omega}{2}\, d\omega}{\int_{\omega=0}^{\omega_0} \frac{\sin\omega}{2}\, d\omega} \tag{7-4}$$

where

$$\overline{AB} = 2\sqrt{R_c^2 - R_H^2 \sin^2\omega}$$

(3) $\overline{BC}$ hydrogen path

$$\xi_{BC,i} = \frac{\int_{\omega=0}^{\omega_0} \exp\left[-\Sigma_{t_i}^{H} \times \overline{BC}\right] \frac{\sin\omega}{2}\, d\omega}{\int_{\omega=0}^{\omega_0} \frac{\sin\omega}{2}\, d\omega} \tag{7-5}$$

where

$$\overline{BC} = \sqrt{R_p^2 - R_H^2 \sin^2\omega} - \sqrt{R_c^2 - R_H^2 \sin^2\omega}$$

(4) $\overline{SQ}$ hydrogen path

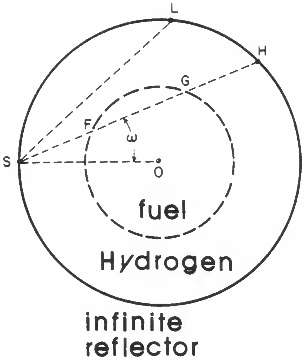
$$\xi_{SQ,i} = \frac{\int_{\omega=\omega_0}^{\pi} \exp\left[-\Sigma_{t_i}^{H} \times \overline{SQ}\right] \frac{\sin\omega}{2}\, d\omega}{\int_{\omega=\omega_0}^{\pi} \frac{\sin\omega}{2}\, d\omega} \tag{7-6}$$

where

$$\overline{SQ} = R_H \cos\omega + \sqrt{R_p^2 - R_H^2 \sin^2\omega} \; .$$

Third point of view: Reflector secondary neutron sources

For the reflector neutron sources there are four types of relevant transmission coefficients. They involve: (1) when a neutron crosses the hydrogen-fuel-hydrogen region, and (2) when a neutron crosses only the hydrogen region. With the aid of Figure 7-5, these transmission coefficient evaluations can be comprehended.



$\overline{OS} = R_p$, $\overline{OG} = R_c$

Thus,

$$\overline{SL} = 2\, R_p \cos \omega$$

$$\overline{SF} = R_p \cos \omega - \sqrt{R_c^2 - R_p^2 \sin^2 \omega}$$

$$\overline{GH} = \overline{SF}$$

$$\overline{FG} = 2 \sqrt{R_c^2 - R_p^2 \sin^2 \omega}$$

for $0 \le \omega \le \pi/2$

Figure 7-5. Schematic drawing for the transmission coefficient evaluation under reflector secondary neutron source point of view for the three-region gas core reactor problem.

The relevant transmission coefficients are given by

(1) $\overline{SF}$ hydrogen path

$$\eta_{SF,i} = \frac{\int_{\omega=0}^{\omega_f} \exp\left[-\, \Sigma_{t_i}^{H} \times \overline{SF}\,\right] \frac{\sin \omega}{2}\, d\,\omega}{\int_{\omega=0}^{\omega_f} \frac{\sin \omega}{2}\, d\,\omega} \tag{7-7}$$

where

$$\overline{SF} = R_p \cos \omega - \sqrt{R_c^2 - R_p^2 \sin^2 \omega} \;\; \text{and} \;\; \omega_f = \sin^{-1}\left[\frac{R_c}{R_p}\right]$$

(2) FG fuel path

$$\eta_{FG,i} = \frac{\int_{\omega=0}^{\omega_f} \exp\left[-\ \Sigma_{ti}^{F}\ x\ FG\right] \frac{\sin\omega}{2}\, d\,\omega}{\int_{\omega=0}^{\omega_f} \frac{\sin\omega}{2}\, d\,\omega} \tag{7-8}$$

where

$$FG = 2\sqrt{R_c^2 - R_p^2 \sin^2\omega}$$

(3) GH hydrogen path

$$\eta_{GH,i} = \frac{\int_{\omega=0}^{\omega_f} \exp\left[-\ \Sigma_{ti}^{H}\ x\ GH\right] \frac{\sin\omega}{2}\, d\,\omega}{\int_{\omega=0}^{\omega_f} \frac{\sin\omega}{2}\, d\,\omega} \tag{7-9}$$

where

$$GH = R_p \cos\omega - \sqrt{R_c^2 - R_p^2 \sin^2\omega}$$

(4) SL hydrogen path

$$\eta_{SL,i} = \frac{\int_{\omega=\omega_f}^{\pi/2} \exp\left[-\ \Sigma_{ti}^{F}\ x\ SL\right] \frac{\sin\omega}{2}\, d\,\omega}{\int_{\omega=\omega_f}^{\omega_f} \frac{\sin\omega}{2}\, d\,\omega} \tag{7-10}$$

where

$$SL = 2\ R_p \cos\omega$$

In Table 7-4 are the transmission coefficient values for the fuel, hydrogen, and reflector secondary neutron sources.

TABLE 7-4. TRANSMISSION COEFFICIENTS FOR ALL SECONDARY NEUTRON SOURCES OF THE THREE-REGION GAS CORE REACTOR PROBLEM

	Four-Group Albedo Theory Method			
	i = 2	i = 2	i = 3	i = 4
ζ_i	.98308	.96821	.94542	.77782
e_i	.90913	.96815	.94288	.92631
$\xi_{SX,i}$	.99411	.97722	.99007	.94699
$\xi_{AB,i}$	.97156	.94675	.90913	.64883
$\xi_{AH,i}$	.99087	.96457	.93658	.91828
$\xi_{AG,i}$	.96486	.96109	.93092	.91147
$\eta_{SF,i}$	.99087	.96109	.93092	.91892
$\eta_{FG,i}$	.97110	.94590	.90771	.64408
$\eta_{GH,i}$	.99087	.96486	.93092	.91147
$\eta_{SL,i}$	.98470	.94198	.89786	.86982

- Fuel radius = R_c = 100.0 cm
- Hydrogen thickness = $R_p - R_c$ = 50.0 cm
- Infinite beryllium oxide reflector
- $R_o = R_c / \sqrt[3]{2}$
- $R_H = \left[\frac{R_p^3 + R_c^3}{2}\right]^{1/3}$

Dissection of P_o and P

As was done for the two-region gas core reactor problem, it is of interest to evaluate: (1) the fraction of neutrons which, having been produced by fission in the fuel region, never traverse the fuel-hydrogen interface, and are absorbed as group i; here denoted by $A^F_{o_i}$, (2) the fraction of neutrons which are produced by fission in the fuel region, spend time out of the fuel region, but are eventually absorbed by nuclei of the fuel as group i; here denoted by A^F_i , and (3) the fraction of neutrons which are produced by fission in the fuel region and are absorbed by hydrogen nuclei as group i; here denoted by A^H_i. In Table 7-5 are the nonleakage probabilities $A^F_{o_i}$, A^F_i , and A^H_i where

$$P^F_o = \sum_{i=1}^{4} A^F_{o_i} \, , \; P^F = \sum_{i=1}^{4} A^F_i \text{ and } P^F_{TNL} = P^F_o + P^F \, .$$

TABLE 7-5. NONLEAKAGE PROBABILITY RESULTS OF THE THREE-REGION GAS CORE REACTOR PROBLEM USING THE FOUR-GROUP ALBEDO THEORY METHOD

	Four-Group Albedo Theory Method			
	i = 1	i = 2	i = 3	i = 4
$A^F_{o_i}$	.84746 E-03	.46700 E-03	.14235 E-05	.16362 E-08
A^F_i	.15303 E-03	.10442 E-02	.15243 E-01	.45393
A^H_i	.00000	.00000	.51114 E-04	.38556 E-02

- Fuel region radius = R_c = 100.0 cm
- Hydrogen region thickness = $R_p - R_c$ = 50.0 cm
- Infinite beryllium oxide reflector
- $R_o = \dfrac{R_c}{\sqrt[3]{2}}$ and $R_H = \left[\dfrac{R_p^3 + R_c^3}{2}\right]^{1/3}$

From Table 7-5, the fuel total principal nonleakage probability, P_o^F, is found to be very small, .17815 E-02, while the fuel total secondary nonleakage probability, P^F, is very high, .46991. It is clear that P^F represents about 100% of the fuel total nonleakage probability value for this three-region gas core reactor problem. The hydrogen total nonleakage probability, P_{TNL}^H, value is found to be very small, .39067 E-02. The hydrogen total nonleakage probability is given by

$$P_{TNL}^H = \sum_{i=1}^{4} A_i^H \quad .$$

Effective Neutron Multiplication Factor, k_{eff}

The neutrons produced by the (n, 2n) interaction in the fuel region are accounted for in k_{eff} value, i.e.,

$$k_{eff} = \sum_{i=1}^{4} \left(A_{o_i}^F + A_i^F\right) \frac{\nu_i \, \Sigma_{f_i} + 2 \, \Sigma_{n,2n_i}}{\Sigma_{a_i}} \quad .$$

Using the numerical values for this problem, k_{eff} = .95437. The program ALB4 used for these numerical calculations is in Appendix G.

Analyses of Sensitivity

Because of its location between the fuel region and the reflector, the hydrogen in the reactor cavity region notably affects neutron multiplication due to both neutron absorption and scattering collisions. Comparative neutronic calculations are performed on the three-region gas core reactor problem in one dimension to determine the impact of the variation of the hydrogen density factor.

Hydrogen Limiting Density Factor

At constant fuel mass and radius, in order to characterize the reference reactor configuration the effective neutron multiplication factor is calculated by employing hydrogen at different density factors (D.F.) in the cavity of the reactor. The hydrogen limiting density factor (D.F.*) corresponds to a maximum in k_{eff}. The variation of k_{eff} is shown in Table 7-6.

TABLE 7-6. EFFECTS OF THE HYDROGEN DENSITY FACTOR VARIATION ON THE THREE-REGION GAS CORE REACTOR PROBLEM PROPERTIES USING THE FOUR-GROUP ALBEDO METHOD

Hydrogen D.F.	Four-Group Albedo Theory Method		
	k_{eff}	P^{H}_{TNL}	P^{F}_{TNL}
1.0	.95437	.39067 E-02	.48108
2.0	.95951	.78195 E-02	.47417
8.5	.96381	.11753 E-01	.48820
1.0	.96713	.15719 E-01	.47791
8.5	.97007	.19730 E-01	.47921
8.5	.97211	.23795 E-01	.48016
8.5	.97348	.27926 E-01	.48078
8.5	.97421	.32130 E-01	.48109
2.0	.97433	.33834 E-01	.48111
8.5	.97435	.34263 E-01	.48111
8.5	.97435	.34692 E-01	.48110
8.7*	.97435	.35984 E-01	.48110
8.5	.97435	.35552 E-01	.48109
8.9	.97434	.35984 E-01	.48108
1.0	.97433	.36416 E-01	.48107
11.0	.97386	.19730 E-01	.48078
11.0	.97283	.45265 E-01	.48020
12.0	.97126	.49842 E-01	.47936
13.0	.96917	.54529 E-01	.47826
14.0	.96659	.59332 E-01	.47693
15.0	.96353	.64257 E-01	.47535

- Fuel region radius = R_c = 100.0 cm
- Hydrogen region thickness = $R_p - R_c$ = 50.0 cm
- Infinite beryllium oxide reflector

- $R_o = R_c / \sqrt[3]{2}$ and $R_H = \left[(R_p^3 + R_c^3) / 2\right]^{1/3}$

Table 7-6 shows that the highest k_{eff} value is obtained for a hydrogen density factor = 8.7 and neutron absorption in hydrogen varies almost proportionately with hydrogen density factor. Illustrated in Figure 7-6 is the hydrogen limiting density factor for the reference reactor configuration (D.F.* = 8.7).

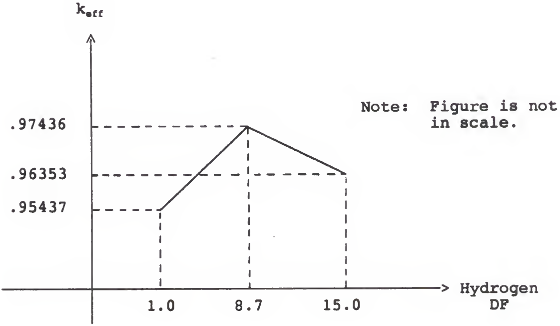


Figure 7-6. Hydrogen limiting density factor for the three-region gas core reactor problem.

<u>Oscillations of the Fuel-Hydrogen Boundary</u>

As a particular example of what can be accomplished using the multigroup albedo theory ideas, consider the problem of predicting and understanding the reactivity (and, thus neutronic feedback) variation due to perturbation of the fuel-hydrogen boundary (e.g., the boundary shape shown as a dashed line in Figure 7-7).

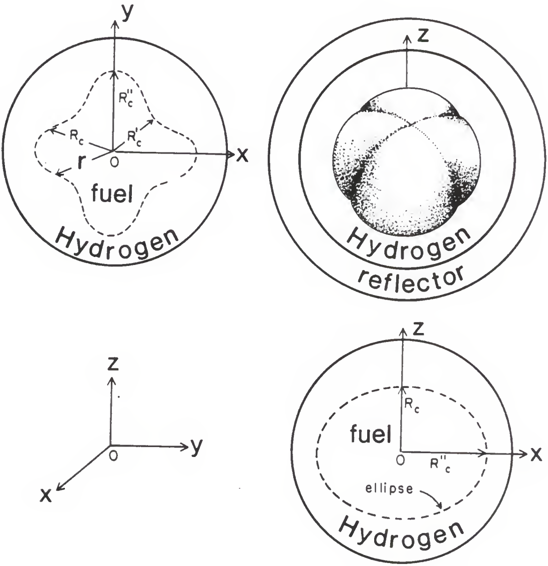


Figure 7-7. Schematic drawing showing a particular oscillation of the fuel-hydrogen boundary of the three-region gas core reactor problem.

The various possible modes of the three-dimensional fuel-hydrogen boundary of the three-region gas core reactor problem cannot be effectively handled by conventional neutronic calculation methods such as XSDRNPM.

Transmission coefficient values for the fuel, hydrogen, and reflector secondary neutron sources

The variation of all forty transmission coefficient values generated by such standing waves of the fuel-hydrogen boundary is an algebraic-geometric exercise that can be solved by using: (1) a plastic mock-up to obtain the length and solid angle values required to estimate the transmission coefficients, and (2) an analytical approach that for complex standing wave geometry can be tougher than the mock-up solution. After the evaluation of the transmission coefficients, the perturbed fuel-hydrogen boundary reactor problem is performed exactly as the one-dimensional three-region gas core reactor.

k_{eff} variation due to a perturbed fuel-hydrogen boundary

For convenience, the two-dimensional standing wave perturbation of the fuel-hydrogen boundary in Figure 7-7 can be described by the magnitude position vector function

$$r = R_c + (R_c^* - R_c) \cos\left(\frac{2\pi\ell}{\lambda}\right) \qquad (7\text{-}11)$$

where λ is the wavelength of the surface wave, R_c" is its maximum amplitude and ℓ is the arc length measured on the circle of radius R_c. It is assumed that

(1) $\dfrac{R_c'' - R_c}{R_c} = \dfrac{R_c - R_c'}{R_c}$ variation is small where R_c' is the wave minimum amplitude, and

(2) the wavelength of the wave is given by $\lambda = \frac{\pi}{2} R_c$.

The resulting changes in k_{eff} values by varying $\left(R_c'' - R_c\right)$ is shown in Table 7-7.

TABLE 7-7. EFFECTS DUE TO OSCILLATING FUEL-HYDROGEN BOUNDARY ON THE THREE-REGION GAS CORE REACTOR PROBLEM PROPERTIES USING THE FOUR-GROUP ALBEDO METHOD

$\dfrac{R_c'' - R_c}{R_c} \times 100\%$	Four-Group Albedo Theory Method		
	k_{eff}	P^H_{TNL}	P^F_{TNL}
2.0	.95437	.39067 E-02	.47169
1.0	.95083	.39427 E-02	.46994
2.0	.95101	.39411 E-02	.47003
3.0	.95121	.39395 E-02	.47013

- R_c = 100.0 cm
- R_p = 150.0 cm
- Infinite beryllium oxide reflector
- $R_o = R_c \,/\, \sqrt[3]{2}$ and $R_H = \left[\left(R_p^3 + R_c^3\right) / 2\right]^{1/3}$

Table 7-7 shows that for $1\% \le \dfrac{R_c'' - R_c}{R_c} \le 3\%$ the k_{eff} values are smaller than the k_{eff} result obtained for the unperturbed spherical fuel-hydrogen boundary case. This result is

due to the increase of the fuel transmittance and consequent reduction in the absorption rate of neutrons by the nuclei of the fuel region.

Nonuniform Atomic Density Distribution in the Fuel Region

For the previous neutronic calculations performed on the gas core reactor problems the gaseous fuel was assumed to be uniformly distributed within the fuel region. Assuming that the total fuel mass is constant, a nonuniform atomic density distribution in the fuel region is used to determine the impact of such behavior on the value of k_{eff}.

Linear atomic density distribution

A linear atomic density distribution, $\rho(r)$, is assumed and given by the expression

$$\rho(r) = \rho_o' (1 + A\, r)$$

where

A = the angular coefficient and

r = the position vector magnitude

The fuel mass conservation condition is

$$\int_{r=0}^{R_c} \rho_o' (1 + A\, r)\, 4\pi\, r^2\, d\, r = \int_{r=0}^{R_c} \rho_o\, 4\pi\, r^2\, d\, r \qquad (7\text{-}12)$$

where

ρ_o = the initial uniform atomic density and

R_c = the radius of the fuel region.

Equation 7-12 reduces to

$$\rho_o' = \frac{\rho_o}{1 + A\,\frac{3R_c}{4}} \,, \qquad (7\text{-}13)$$

thus,

$$\rho(r) = \frac{\rho_o}{1 + A\,\frac{3R_c}{4}} (1 + A\,r) \quad . \qquad (7\text{-}14)$$

The corresponding fuel macroscopic cross section, $\Sigma(r)$, is given by

$$\Sigma(r) = \frac{1 + A\,r}{1 + A\,\frac{3R_c}{4}} \Sigma_o \qquad (7\text{-}15)$$

where

Σ_o = the fuel macroscopic cross section for the initial uniform atomic density.

Fuel secondary source position

As done previously, it is assumed that the neutron sources are isotropic and are homogeneously distributed at the surface of a sphere of radius R_o given by the equation

$$\int_{r=0}^{R_o} \rho_o' \,(1 + A\,r)\, 4\pi\, r^2\, d\,r = \int_{r=r_o}^{R_c} \rho_o' \,(1 + A\,r)\, 4\pi\, r^2\, d\,r \,, \qquad (7\text{-}16)$$

i.e., this imaginary sphere contains half of the total fuel mass. Thus, R_o must satisfy the quartic equation given by

$$R_o^4 + \frac{4}{3A} R_o^3 - \frac{2R_c^3}{A}\left(\frac{1}{3} + A\,\frac{R_c}{4}\right) = 0 \tag{7-17}$$

and

$$0 < R_o < R_c \quad .$$

<u>Cases investigated</u>: $A = \pm\,0.01$

Figure 7-8 illustrates the variation of $\rho(r)$ and $\Sigma(r)$ in the fuel region for the cases $A = \pm 0.01$.

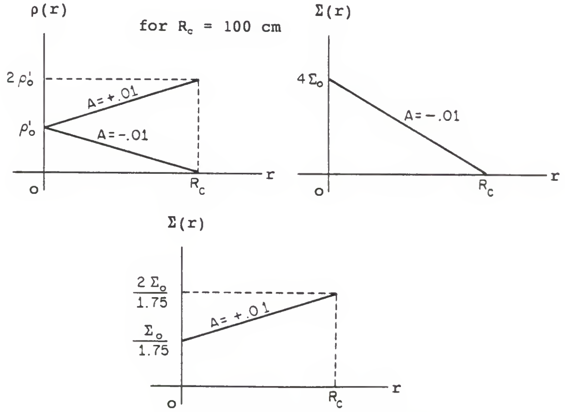


Figure 7-8. Qualitative graph of $\rho(r)$ and $\Sigma(r)$ in the fuel region of a three-region gas core reactor problem.

The resulting k_{eff} variation is shown in Table 7-8.

TABLE 7-8. EFFECTS DUE TO NONUNIFORM ATOMIC DENSITY DISTRIBUTION IN THE FUEL REGION ON THE THREE-REGION GAS CORE REACTOR PROBLEM USING THE FOUR-GROUP ALBEDO METHOD

A	Four-Group Albedo Theory Method			
	k_{eff}	P^{H}_{TNL}	P^{FUEL}_{TNL}	R_o
- 0.01	1.1231	.27303 E-02	.55867	61.43
0.00	.95437	.39067 E-02	.46956	79.37
+ 0.01	.90812	.42273 E-02	.44840	81.58

- R_c = 100 cm
- R_p = 150 cm
- $R_H = \left[(R_p^3 + R_c^3) / 2\right]^{1/3}$
- R_o in cm

The results in Table 7-8 indicate that relative to a uniform density core A = -0.01 yields an increase in k_{eff}, while A = +0.01 yields a decrease in k_{eff}. It appears that the required variation of R_o yields effects on core transmittance which completely explain these results.

The program ALB4 employed for these numerical calculations is in Appendix G.

CHAPTER VIII
CONCLUSIONS AND RECOMMENDATIONS

Introduction

In this dissertation is described a multigroup neutron albedo theory method is described which can be used to perform neutronic calculations for thermal nuclear reactors. In the neutron albedo method, description of average neutron histories is accomplished through the specification of reflection and transmission probabilities at each neutron traversal of the core-reflector interface. For each energy group and for energy group transfers, these neutron albedo parameters are expressed in terms of the cross sections and geometry of the core and the reflector. The effective neutron multiplication factor and reactivity can be expressed as analytical functions of the neutron albedos, and thereby, as direct functions of cross sections and geometry. Thus, the neutron albedo theory method is an analytical method that does not try to solve an explicit describing equation for the neutron flux but uses such relations to obtain partial information that is combined with the average behavior of the neutrons in the physical system to obtain nonleakage probability values and effective and infinite neutron multiplication factors. Therefore, the

neutron albedo theory methodology is very different from the methodology used in conventional discrete ordinate computer codes such as XSDRNPM.

Final Conclusion

The primary goal of developing a multigroup neutron albedo method has been achieved herein. Moreover, application to neutronic calculations such as nonleakage probabilities and effective neutron multiplication factors, and investigation of the main parameters of the four-group albedo method have been accomplished with the aid of three calculational reactor problems. In Chapters V, VI, and VII of this work, the four-group albedo theory is used to calculate the total nonleakage probability and effective neutron multiplication factor for reactors with large and small optical-dimension cores displaying the matrix of parameter dissection of the total principal and secondary nonleakage probabilities. In Table 8-1 is shown the importance of the total principal and secondary nonleakage probabilities for the analyzed large and small optical-path-length (O.P.L.) core (two-region) reactor problems.

TABLE 8-1. LARGE VERSUS SMALL OPTICAL-PATH-LENGTH CORE (TWO-REGION) REACTOR PROBLEM VALUES USING THE FOUR-GROUP ALBEDO METHOD

	Four-Group Albedo Method	
	Large O.P.L.	Small O.P.L.
P_o	.89433	.92404 E-03
P	.46129 E-01	.53460
$P_{TNL} = P_o + P$	.94046	.53553
P_o/P_{TNL}	~ 95.1%	0.2%
P/P_{TNL}	~ 4.9%	99.8%

From Table 8-1, the extremely important role of the reflector-moderator in small O.P.L. gas core concepts is demonstrated. The total secondary nonleakage probability, P, represents approximately 100% of the total nonleakage probability, P_{TNL} , for such cores.

Unfortunately, the multigroup neutron albedo method usually depends on assumptions that make the final solution less accurate than the more rigorous solution methods (such as XSDRNPM). Nevertheless, for many cases (such as gas core reactor problems), if the approximate solution is reasonably acceptable, the loss in accuracy and generality is more than outweighed by the increased simplicity in the calculations, the resultant reduction in computer time, and especially the clarity of identification of various physical phenomena influences. What is sought here is the development of a simplified approach to neutronic problems under the neutron albedo point of view with a reasonably quick and inexpensive

method that is clearly more understandable than conventional neutron transport methods. In order to quickly see relations among the albedos, transmittances, nonleakage probabilities, and the effective neutron multiplication factor, the multigroup neutron albedo method represents a powerful and helpful tool.

Recommendations for Future Work

The significant amount of research performed on the small optical-dimension core reactors (two- and three-region), as presented in this dissertation, has furnished some insights and intuitive procedures. Future research should focus on developing analytical and intuitive methods of determining the core transmission coefficient values which actually constitute the key item for accurate effective neutron multiplication factor results.

Application of the albedo approach to the determination of neutronic-fluid dynamic coupling in gas core reactor concepts [15] could be an important practical application that is uniquely suited to the method. Such an effort, if successful, would certainly justify, on practical grounds, the considerations and ideas developed in this dissertation.

APPENDIX A
ALGORITHM FOR ${}^{m}S_i$

The following algorithm for calculating ${}^{m}S_i$ is developed employing the known sources of initial leakage of m-1 group albedo theory, ${}^{m-1}S_i$.

Initial step: Evaluation of ${}^{m}S_i$ $(i < m)$

Using the assumed known expression of ${}^{m-1}S_i$, replace the superscript (m-1) by the superscript m; thus,

$$ {}^{m}S_i = {}^{m-1}S_i \ (m - 1 \rightarrow m). \tag{A-1} $$

Final step: Evaluation of ${}^{m}S_m$

$$ {}^{m}S_m = (1 - P_{o_m}) \left(\chi_m + \sum_{i=1}^{m-1} \frac{{}^{m}S_i}{1 - P_{o_i}} \, P_{o_i} \, \frac{\Sigma_{S_{im}}}{\Sigma_{R_i}} \right) . \tag{A-2} $$

APPENDIX B
TWO-GROUP PARTIAL SECONDARY NONLEAKAGE PROBABILITY DERIVATION

Here Equation 2-11 is derived under the point of view of two-group albedo theory. The total secondary nonleakage probability, P, is a sum of two separate processes. Specifically,

(1) The probability that a neutron produced by fission in the core leaks as thermal (i=2) and is absorbed in the core, here denoted by ${}^2P_2$, and

(2) The probability that a neutron produced by fission in the core leaks as fast (i=1) neutron and is absorbed in the core, here denoted by ${}^2P_1$.

Hence, one can write $P = {}^2P_1 + {}^2P_2$ or $P = {}^2S_1\ {}^2Q_1 + {}^2S_2\ {}^2Q_2$ where ${}^2Q_1$ is the probability that if one neutron leaks as a group 1 neutron from the core, it will be absorbed in the core and ${}^2Q_2$ is the probability that if one neutron leaks as a group 2 neutron from the core, it will be absorbed in the core. ${}^2S_1$ and ${}^2S_2$ values are shown in Figure 2-1.

First, with the aid of Figure B-1, the ${}^2Q_2$ expression is determined.

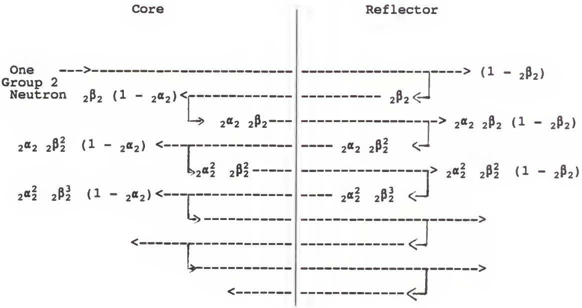


Figure B-1. Schematic drawing showing the two-group albedo approach strategy for determining ${}^2Q_2$ expression.

In the passage of group 2 neutrons from the core to the reflector, ${}_2\beta_2$ is the probability that group 2 neutrons incident on the reflector from the core will be reflected with group 2 energy. Similarly, ${}_2\alpha_2$ is the probability that group 2 neutrons incident on the core from the reflector will be reflected with group 2 energy. From Figure B-1, the ${}^2Q_2$ is given by

$$\begin{aligned}
{}^2Q_2 &= {}_2\beta_2\left(1 - {}_2\alpha_2\right) + {}_2\alpha_2\ {}_2\beta_2^2\left(1 - {}_2\alpha_2\right) + {}_2\alpha_2^2\ {}_2\beta_2^3\left(1 - {}_2\alpha_2\right) + \cdots = \\
&= {}_2\beta_2\left(1 - {}_2\alpha_2\right)\left[1 + {}_2\alpha_2\ {}_2\beta_2 + \left({}_2\alpha_2\ {}_2\beta_2\right)^2 + \cdots\right] = \\
&= \frac{\left(1 - {}_2\alpha_2\right)\ {}_2\beta_2}{1 - {}_2\alpha_2\ {}_2\beta_2}
\end{aligned} \tag{B-1}$$

From Figure B-1 and Equation B-1, one determines the probability that if one neutron leaks as a group 2 neutron from the reflector, it will be absorbed in the reflector, here denoted by ${}^2\bar{Q}_2$. Thus,

$$ {}^2\bar{Q}_2 = \frac{(1 - {}_2\beta_2)\ {}_2\alpha_2}{1 - {}_2\alpha_2\ {}_2\beta_2} \tag{B-2} $$

and

$$ 1 - {}^2\bar{Q}_2 = \frac{1 - {}_2\alpha_2}{1 - {}_2\alpha_2\ {}_2\beta_2} \tag{B-3} $$

represents the probability that if one neutron leaks as group 2 neutron from the reflector, it will be absorbed in the core.

Finally, with the aid of Figure B-2, one determines the ${}^2Q_1$ expression.

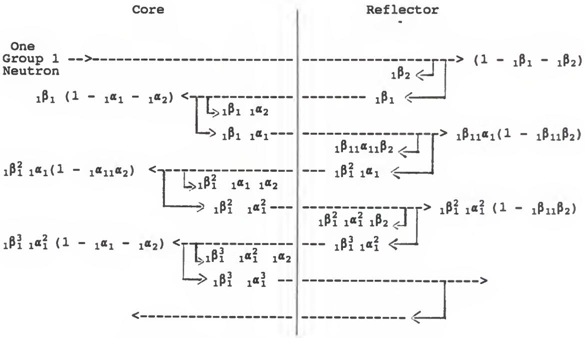


Figure B-2. Schematic drawing showing the two-group albedo approach strategy for determining ${}^2Q_1$ expression.

In the passage of group 1 neutrons from the core to reflector, ${}_1\beta_1$ is the probability that group 1 neutrons incident on the reflector from the core will be reflected with group 1 energy and ${}_1\beta_2$ is the probability that group 1 neutrons incident on the reflector from the core will be reflected with group 2 energy. There are similar definitions for ${}_1\alpha_1$ and ${}_1\alpha_2$. From Figure B-2 and Equations B-1 and B-3, the ${}^2Q_1$ equation is obtained given by

$$
\begin{aligned}
{}^2Q_1 = & \left[\, {}_1\beta_1 (1 - {}_1\alpha_1 - {}_1\alpha_2) + {}_1\beta_1^2\, {}_1\alpha_1 (1 - {}_1\alpha_1 - {}_1\alpha_2) + {}_1\beta_1^3\, {}_1\alpha_1^2 (1 - {}_1\alpha_1 - {}_1\alpha_2) + \cdots \right] + \\
& \left[\, {}_1\beta_2 + {}_1\beta_1\, {}_1\alpha_1\, {}_1\beta_2 + {}_1\beta_1^2\, {}_1\alpha_1^2\, {}_1\beta_2 + \cdots \right] \left(1 - {}^2\overline{Q}_2\right) + \\
& \left[\, {}_1\beta_1\, {}_1\alpha_2 + {}_1\beta_1^2\, {}_1\alpha_1\, {}_1\alpha_2 + {}_1\beta_1^3\, {}_1\alpha_1^2\, {}_1\alpha_2 + \cdots \right] {}^2Q_2 = \\
= & \frac{{}_1\beta_1 (1 - {}_1\alpha_1 - {}_1\alpha_2)}{1 - {}_1\beta_1\, {}_1\alpha_1} + \frac{{}_1\beta_2}{1 - {}_1\beta_1\, {}_1\alpha_1} \cdot \frac{1 - {}_2\alpha_2}{1 - {}_2\alpha_2\, {}_2\beta_2} \\
& + \frac{{}_1\beta_1\, {}_1\alpha_2}{1 - {}_1\alpha_1\, {}_1\beta_1} \cdot \frac{(1 - {}_2\alpha_2)\, {}_2\beta_2}{(1 - {}_2\alpha_2\, {}_2\beta_2} = \\
= & \frac{(1 - {}_1\alpha_1)\, {}_1\beta_1}{1 - {}_1\alpha_1\, {}_1\beta_1} - \frac{{}_1\beta_1\, {}_1\alpha_2}{1 - {}_1\alpha_1\, {}_1\beta_1} \cdot \frac{1 - {}_2\beta_2}{1 - {}_2\alpha_2\, {}_2\beta_2} \\
& + \frac{{}_1\beta_2}{1 - {}_1\alpha_1\, {}_1\beta_1} \cdot \frac{1 - {}_2\alpha_2}{1 - {}_2\alpha_2\, {}_2\beta_2} \ .
\end{aligned}
\tag{B-4}
$$

Equations B-1 and B-4 represent the desired result of the probabilities ${}^2Q_2$ and ${}^2Q_1$, respectively.

APPENDIX C
FOUR-GROUP CORE ALBEDO DERIVATION

Here the four-group core albedos are derived for a spherical core of radius R. All group constants refer to the homogeneous core.

Determination of ${}_1\alpha_1$, ${}_1\alpha_2$, ${}_1\alpha_3$, and ${}_1\alpha_4$

The governing equations and the boundary conditions for the determination of ${}_1\alpha_1$, ${}_1\alpha_2$, ${}_1\alpha_3$, and ${}_1\alpha_4$ are

$$- D_1 \nabla^2 \phi_1 + \Sigma_{R_1} \phi_1 = 0$$

$$- \Sigma_{S_{12}} \phi_1 - D_2 \nabla^2 \phi_2 + \Sigma_{R_2} \phi_2 = 0$$

$$- \Sigma_{S_{13}} \phi_1 - \Sigma_{S_{23}} \phi_2 - D_3 \nabla^2 \phi_3 + \Sigma_{R_3} \phi_3 = 0$$

$$- \Sigma_{S_{14}} \phi_1 - \Sigma_{S_{24}} \phi_2 - \Sigma_{S_{34}} \phi_3 - D_4 \nabla^2 \phi_4 + \Sigma_{R_4} \phi_4 = 0$$

at $r = 0$, ϕ_1, ϕ_2, ϕ_3, and ϕ_4 are finite and

at $r = R$, $J_-\,]_{i=1} = 1$, $J_-\,]_{i=2,3,4} = 0$.

Finally, at $r = R$, ${}_1\alpha_1 = \dfrac{J_+\,]_{i=1}}{J_-\,]_{i=1}}$,

${}_1\alpha_2 = \dfrac{J_+\,]_{i=2}}{J_-\,]_{i=1}}$, ${}_1\alpha_3 = \dfrac{J_+\,]_{i=3}}{J_-\,]_{i=1}}$, and ${}_1\alpha_4 = \dfrac{J_+\,]_{i=4}}{J_-\,]_{i=1}}$.

Determination of ${}_2\alpha_2$, ${}_2\alpha_3$, and ${}_2\alpha_4$

The governing equations and the boundary conditions for the determination of ${}_2\alpha_2$, ${}_2\alpha_3$, and ${}_2\alpha_4$ are

$$- D_2 \nabla^2 \phi_2 + \Sigma_{R_2} \phi_2 = 0$$

$$- \Sigma_{S_{23}} \phi_2 - D_3 \nabla^2 \phi_3 + \Sigma_{R_3} \phi_3 = 0$$

$$- \Sigma_{S_{24}} \phi_2 - \Sigma_{S_{34}} \phi_3 - D_4 \nabla^2 \phi_4 + \Sigma_{R_4} \phi_4 = 0$$

at $r = 0$, ϕ_2, ϕ_3, and ϕ_4 are finite and

at $r = R$, $J_-\,]_{i=2} = 1$, $J_-\,]_{i=3,4} = 0$.

Finally, at $r = R$, ${}_2\alpha_2 = \dfrac{J_+\,]_{i-2}}{J_-\,]_{i-2}}$,

${}_2\alpha_3 = \dfrac{J_+\,]_{i-3}}{J_-\,]_{i-2}}$, and ${}_2\alpha_4 = \dfrac{J_+\,]_{i-4}}{J_-\,]_{i-2}}$.

Determination of ${}_3\alpha_3$ and ${}_3\alpha_4$

The governing equations and the boundary conditions for the determination of ${}_3\alpha_3$ and ${}_3\alpha_4$ are

$$- D_3 \nabla^2 \phi_3 + \Sigma_{R_3} \phi_3 = 0$$

$$- \Sigma_{S_{34}} \phi_3 - D_4 \nabla^2 \phi_4 + \Sigma_{R_4} \phi_4 = 0$$

at $r = 0$, ϕ_3 and ϕ_4 are finite and

at $r = R$, $J_- \,]_{i=3} = 1$, $J_- \,]_{i=4} = 0$.

Finally, at $r = R$, ${}_3\alpha_3 = \dfrac{J_+ \,]_{i-3}}{J_- \,]_{i-3}}$ and ${}_3\alpha_4 = \dfrac{J_+ \,]_{i-4}}{J_- \,]_{i-3}}$.

Determination of ${}_4\alpha_4$

The governing equation and the boundary conditions for the determination of ${}_4\alpha_4$ are

$$- D_4 \nabla^2 \phi_4 + \Sigma_{R_4} \phi_4 = 0$$

at $r = 0$, ϕ_4 is finite and at $r = R$, $J_- \,]_{i=4} = 1$.

Finally, at $r = R$, ${}_4\alpha_4 = \dfrac{J_+ \,]_{i-4}}{J_- \,]_{i-4}}$.

The analytical expressions of the core albedos (${}_i\alpha_{i'}$) are given below.

${}_i\alpha_i$ (i = 1, 2, 3, 4)

$$ {}_i\alpha_i = \frac{1 - 2\,D_i\left[L_i \coth(L_iR) - \frac{1}{R}\right]}{1 + 2\,D_i\left[L_i \coth(L_iR) - \frac{1}{R}\right]} \tag{C-1} $$

where

$$ L_i = \sqrt{\frac{\Sigma_{R_i}}{D_i}}\,. $$

${}_i\alpha_{i+1}$ (i = 1, 2, 3)

$$ {}_i\alpha_{i+1} = \frac{4\,\Sigma_{s_{i,i+1}}}{L_i^2 - L_{i+1}^2} \cdot \frac{L_i \coth(L_i R) - L_{i+1}\coth(L_{i+1} R)}{\left[1 + 2\,D_i\left[L_i \coth(L_i R) - \frac{1}{R}\right]\right]\left[1 + 2\,D_{i+1}\left[L_{i+1}\coth(L_{i+1} R) - \frac{1}{R}\right]\right]} \tag{C-2} $$

$_{i}\alpha_{i+2}$ (i = 1, 2)

$$_{i}\alpha_{i+2} = \frac{1}{2\,R}\left[E_{i3} \sinh(L_{i+2}\,R) + F_{i2} \sinh(L_i\,R) + F_{i3} \sinh(L_{i+1}\,R) \right]$$

where

$$E_{i1} = \frac{4\,R}{\left[1 + 2\,D_i\left[L_i \coth(L_i\,R) - \frac{1}{R}\right]\right] \sinh(L_i\,R)}$$

$$F_{i1} = \frac{\Sigma_{s_{i\,i+1}}\, E_{i1}}{D_{i+1}\,(L_{i+1}^2 - L_i^2)}$$

$$E_{i2} = -\frac{F_{i1} \sinh(L_i\,R)\left[1 + 2\,D_{i+1}\left[L_i \coth(L_i\,R) - \frac{1}{R}\right]\right]}{\sinh(L_{i+1}\,R)\left[1 + 2\,D_{i+1}\left[L_{i+1} \coth(L_{i+1}\,R) - \frac{1}{R}\right]\right]}$$

$$F_{i2} = \frac{\Sigma_{s_{i\,i+2}}\, E_{i1} + \Sigma_{s_{i+1\,i+2}}\, F_{i1}}{D_{i+2}\,(L_{i+2}^2 - L_i^2)} \qquad \text{(C-3)}$$

$$F_{i3} = \frac{\Sigma_{s_{i+1\,i+2}}\, E_{i2}}{D_{i+2}\,(L_{i+2}^2 - L_{i+1}^2)} \quad \text{and}$$

$$E_{i3} = -\left[F_{i2} \sinh(L_i\,R)\left[1 + 2\,D_{i+2}\left[L_i \coth(L_i\,R) - \frac{1}{R}\right]\right] + \right.$$

$$\left. F_{i3} \sinh(L_{i+1}\,R)\left[1 + 2\,D_{i+2}\left[L_{i+1} \coth(L_{i+1}\,R) - \frac{1}{R}\right]\right]\right] /$$

$$\left[\sinh(L_{i+2}\,R)\left[1 + 2\,D_{i+2}\left[L_{i+2} \coth(L_{i+2}\,R) - \frac{1}{R}\right]\right]\right] .$$

<u>${}_{1}\alpha_4$ (i = 1)</u>

$$
{}_{1}\alpha_4 = \frac{1}{2\,R}\,[\,E_{i4}\sinh(L_4\,R) + F_{i4}\sinh(L_1\,R) + F_{i5}\sinh(L_2\,R) + F_{i6}\sinh(L_3\,R)\,]
$$

where

$$
F_{i4} = \frac{\Sigma_{s_{14}} E_{i1} + \Sigma_{s_{24}} F_{i1} + \Sigma_{s_{34}} F_{i2}}{D_4\,(L_4^2 - L_1^2)}
$$

$$
F_{i5} = \frac{\Sigma_{s_{24}} E_{i2} + \Sigma_{s_{34}} F_{i3}}{D_4\,(L_4^2 - L_1^2)} \qquad \text{(C-4)}
$$

$$
F_{i6} = \frac{\Sigma_{s_{34}} E_{i3}}{D_4\,(L_4^2 - L_1^2)}\ ,\ \text{and}
$$

$$
\begin{aligned}
E_{i4} = -\Big[\, & F_{i4}\sinh(L_i\,R)\Big[1 + 2\,D_4\Big[L_i\coth(L_1\,R) - \frac{1}{R}\Big]\Big] + \\
& F_{i5}\sinh(L_2\,R)\Big[1 + 2\,D_4\Big[L_2\coth(L_2\,R) - \frac{1}{R}\Big]\Big] + \\
& F_{i6}\sinh(L_3\,R)\Big[1 + 2\,D_4\Big[L_3\coth(L_3\,R) - \frac{1}{R}\Big]\Big]\Big] / \\
& \Big[\sinh(L_4\,R)\Big[1 + 2\,D_4\Big[L_4\coth(L_4\,R) - \frac{1}{R}\Big]\Big]\Big]\ .
\end{aligned}
$$

APPENDIX D
FOUR-GROUP REFLECTOR ALBEDO
DERIVATION FOR A FINITE REFLECTOR

In this appendix, the four-group reflector albedos are derived for a reflector of inner radius R and outer radius H. The reflector thickness is given by the expression H - R.

Determination of ${}_1\beta_1$, ${}_1\beta_2$, ${}_1\beta_3$, and ${}_1\beta_4$

The neutrons produced by the (n, 2n) interaction in the reflector are only accounted for in ${}_1\beta_1$, ${}_1\beta_2$, ${}_1\beta_3$, and ${}_1\beta_4$ values because the (n, 2n) reaction neutrons are born in the first energy group.

The governing equations and the boundary conditions for the determination of ${}_1\beta_1$, ${}_1\beta_2$, ${}_1\beta_3$, and ${}_1\beta_4$ are

$$- D_1 \nabla^2 \phi_1 + \Sigma_{R_1} \phi_1 = 2 \Sigma_{n,2n_1} \phi_1$$

$$- \Sigma_{S_{12}} \phi_1 - D_2 \nabla^2 \phi_2 + \Sigma_{R_2} \phi_2 = 0$$

$$- \Sigma_{S_{13}} \phi_1 - \Sigma_{S_{23}} \phi_2 - D_3 \nabla^2 \phi_3 + \Sigma_{R_3} \phi_3 = 0$$

$$- \Sigma_{S_{14}} \phi_1 - \Sigma_{S_{24}} \phi_2 - \Sigma_{S_{34}} \phi_3 - D_4 \nabla^2 \phi_4 + \Sigma_{R_4} \phi_4 = 0$$

at $r = R$, $J_+]_{i=1} = 1$, $J_+]_{i=2,3,4} = 0$ and

at $r = H$, $J_-]_{i=1,2,3,4} = 0$.

Finally, at r = R, $_1\beta_1 = \dfrac{J_-\,]_{i-1}}{J_+\,]_{i-1}}$,

$_1\beta_2 = \dfrac{J_-\,]_{i-2}}{J_+\,]_{i-1}}$, $_1\beta_3 = \dfrac{J_-\,]_{i-3}}{J_+\,]_{i-1}}$, and $_1\beta_4 = \dfrac{J_-\,]_{i-4}}{J_+\,]_{i-1}}$.

Determination of $_2\beta_2$, $_2\beta_3$, and $_2\beta_4$

The governing equations and the boundary conditions for the determination of $_2\beta_2$, $_2\beta_3$, and $_2\beta_4$ are

$$- D_2 \nabla^2 \phi_2 + \Sigma_{R_2} \phi_2 = 0$$

$$- \Sigma_{S_{23}} \phi_2 - D_3 \nabla^2 \phi_3 + \Sigma_{R_3} \phi_3 = 0$$

$$- \Sigma_{S_{24}} \phi_2 - \Sigma_{S_{34}} \phi_3 - D_4 \nabla^2 \phi_4 + \Sigma_{R_4} \phi_4 = 0$$

at r = R, $J_+\,]_{i=2} = 1$, $J_+\,]_{i=3,4} = 0$ and

at r = H, $J_-\,]_{i=2,3,4} = 0$.

Finally, at r = R, $_2\beta_2 = \dfrac{J_-\,]_{i-2}}{J_+\,]_{i-2}}$,

$_2\beta_3 = \dfrac{J_-\,]_{i-3}}{J_+\,]_{i-2}}$, and $_2\beta_4 = \dfrac{J_-\,]_{i-4}}{J_+\,]_{i-2}}$.

Determination of ${}_3\beta_3$ and ${}_3\beta_4$

The governing equations and the boundary conditions for the determination of ${}_3\beta_3$ and ${}_3\beta_4$ are

$$- D_3 \nabla^2 \phi_3 + \Sigma_{R_3} \phi_3 = 0$$

$$- \Sigma_{S_{34}} \phi_3 - D_4 \nabla^2 \phi_4 + \Sigma_{R_4} \phi_4 = 0$$

at $r = R$, $J_+ \,]_{i=3} = 1$, $J_+ \,]_{i=4} = 0$ and

at $r = H$, $J_- \,]_{i=3,4} = 0$.

Finally, at $r = R$, ${}_3\beta_3 = \dfrac{J_- \,]_{i-3}}{J_+ \,]_{i-3}}$ and ${}_3\beta_4 = \dfrac{J_- \,]_{i-4}}{J_+ \,]_{i-3}}$.

Determination of ${}_4\beta_4$

The governing equations and the boundary conditions for the determination of ${}_4\beta_4$ are

$$- D_4 \nabla^2 \phi_4 + \Sigma_{R_4} \phi_4 = 0$$

at $r = R$, $J_+ \,]_{i=4} = 1$ and

at $r = H$, $J_- \,]_{i=4} = 0$.

Finally, at $r = R$, ${}_4\beta_4 = \dfrac{J_- \,]_{i-4}}{J_+ \,]_{i-4}}$.

The analytical expressions of the reflector albedos ($_i\beta_{i'}$) are given below.

$_i\beta_i$ (i = 1, 2, 3, 4)

$$_i\beta_i = C_{i1} \frac{e^{-k_i R}}{R} \left[\frac{1}{4} - \frac{D_i}{2} (k_i + \frac{1}{R}) \right] + \qquad \text{(D-1)}$$

$$C_{i2} \frac{e^{+k_i R}}{R} \left[\frac{1}{4} + \frac{D_i}{2} (k_i - \frac{1}{R}) \right]$$

where

$$k_i = \sqrt{\frac{\Sigma_{R_i} - 2\,\Sigma_{n,2n_i}}{D_i}},$$

and C_{i1} and C_{i2} are the solution of the following system

$$0 = C_{i1} e^{-k_i H} \left[1 - 2 D_i (k_i + \frac{1}{H}) \right] + C_{i2} e^{+k_i H} \left[1 + 2 D_i (k_i - \frac{1}{H}) \right]$$

$$4R = C_{i1} e^{-k_i R} \left[1 + 2 D_i (k_i + \frac{1}{R}) \right] + C_{i2} e^{+k_i R} \left[1 - 2 D_i (k_i - \frac{1}{R}) \right] .$$

$_i\beta_{i+1}$ (i = 1, 2, 3)

$$_i\beta_{i+1} = \frac{1}{2R}\left[C_{i3}\, e^{-k_{i+1} R} + C_{i4}\, e^{+k_{i+1} R} + G_{i1}\, e^{-k_i R} + G_{i2}\, e^{+k_i R} \right] \qquad \text{(D-2)}$$

where

$$G_{i1} = - \frac{\Sigma_{s_{i i+1}}}{D_{i+1}\,(k_i^2 - k_{i+1}^2)}\, C_{i1} \ ,$$

$$G_{i2} = - \frac{\Sigma_{s_{i i+1}}}{D_{i+1}\,(k_i^2 - k_{i+1}^2)}\, C_{i2} \ ,$$

and C_{i3} and C_{i4} are the solutions of the system

$$- G_{i1}\, e^{-k_i R}\left[1 + 2\, D_{i+1}\, (k_i + R) \right] - G_{i2}\, e^{+k_i R}\left[1 - 2\, D_{i+1}\, (k_i - \frac{1}{R}) \right] =$$

$$C_{i3}\, e^{-k_{i+1} R}\left[1 + 2\, D_{i+1}\, (k_{i+1} + R) \right] + C_{i4}\, e^{+k_{i+1} R}\left[1 - 2\, D_{i+1}\, (k_{i+1} - R) \right]$$

and

$$- G_{i1}\, e^{-k_i H}\left[1 - 2\, D_{i+1}\, (k_i + \frac{1}{H}) \right] - G_{i2}\, e^{+k_i H}\left[1 + 2\, D_{i+1}\, (k_i - \frac{1}{H}) \right] =$$

$$C_{i3}\, e^{-k_{i+1} H}\left[1 - 2\, D_{i+1}\, (k_{i+1} + \frac{1}{H}) \right] + C_{i4}\, e^{+k_{i+1} H}\left[1 + 2\, D_{i+1}\, (k_{i+1} - \frac{1}{H}) \right].$$

$_{i}\beta_{i+2}$ (i = 1, 2)

$$_{i}\beta_{i+2} = \frac{1}{2\,R}\left[C_{i5}\, e^{-k_{i+2} R} + C_{i6}\, e^{+k_{i+2} R} + G_{i3}\, e^{-k_i R} + \right.$$

$$\left. G_{i4}\, e^{+k_i R} + G_{i5}\, e^{-k_{i+1} R} + G_{i6}\, e^{+k_{i+1} R} \right] \tag{D-3}$$

where

$$G_{i3} = -\frac{\Sigma_{s_{i\,i+2}} C_{i1} + \Sigma_{s_{i+1\,i+2}} G_{i1}}{D_{i+2}\,(k_i^2 - k_{i+2}^2)}$$

$$G_{i4} = -\frac{\Sigma_{s_{i\,i+2}} C_{i2} + \Sigma_{s_{i+1\,i+2}} G_{i2}}{D_{i+2}\,(k_i^2 - k_{i+2}^2)}$$

$$G_{i5} = -\frac{\Sigma_{s_{i+1\,i+2}}}{D_{i+2}\,(k_{i+1}^2 - k_{i+2}^2)}\, C_{i3}$$

$$G_{i6} = -\frac{\Sigma_{s_{i+1\,i+2}}}{D_{i+2}\,(k_{i+1}^2 - k_{i+2}^2)}\, C_{i4}\ ,$$

and C_{i5} and C_{i6} are the solutions of the following system

$$- G_{i3}\, e^{-k_i R} \Big[1 + 2\, D_{i+2}\, (k_i + \frac{1}{R}) \Big] - G_{i4}\, e^{+k_i R} \Big[1 - 2\, D_{i+2}\, (k_i - \frac{1}{R}) \Big]$$

$$- G_{i5}\, e^{-k_{i+1} R} \Big[1 + 2\, D_{i+2}\, (k_{i+1} + \frac{1}{R}) \Big] - G_{i6}\, e^{+k_{i+1} R} \Big[1 - 2\, D_{i+2}\, (k_{i+1} - \frac{1}{R}) \Big] =$$

$$C_{i5}\, e^{-k_{i+2} R} \Big[1 + 2\, D_{i+2}\, (k_{i+2} + \frac{1}{R}) \Big] + C_{i6}\, e^{+k_{i+2} R} \Big[1 - 2\, D_{i+2}\, (k_{i+2} - \frac{1}{R}) \Big]$$

and

$$- G_{i3}\, e^{-k_i H} \Big[1 - 2\, D_{i+2}\, (k_i + \frac{1}{H}) \Big] - G_{i4}\, e^{+k_i H} \Big[1 + 2\, D_{i+2}\, (k_i - \frac{1}{H}) \Big]$$

$$- G_{i5}\, e^{-k_{i+1} H} \Big[1 - 2\, D_{i+2}\, (k_{i+1} + \frac{1}{H}) \Big] - G_{i6}\, e^{+k_{i+1} H} \Big[1 + 2\, D_{i+2}\, (k_{i+1} - \frac{1}{H}) \Big] =$$

$$C_{i5}\, e^{-k_{i+2} H} \Big[1 - 2\, D_{i+2}\, (k_{i+2} + \frac{1}{H}) \Big] + C_{i6}\, e^{+k_{i+2} H} \Big[1 + 2\, D_{i+2}\, (k_{i+2} - \frac{1}{H}) \Big] \; .$$

$_{i}\beta_4$ (i = 1)

$$_{i}\beta_4 = \frac{1}{2\,R}\Big[\,C_{i7}\,e^{-k_4 R} + C_{i8}\,e^{+k_4 R} + G_{i7}\,e^{-k_1 R} + G_{i8}\,e^{+k_1 R} +$$

$$G_{i9}\,e^{-k_2 R} + G_{i10}\,e^{+k_2 R} + G_{i11}\,e^{-k_3 R} + G_{i12}\,e^{+k_3 R}\Big] \qquad \text{(D-4)}$$

where

$$G_{i7} = -\,\frac{\Sigma_{s_{14}}\,C_{i1} + \Sigma_{s_{24}}\,G_{i1} + \Sigma_{s_{34}}\,G_{i3}}{D_4\,(k_1^2 - k_4^2)}$$

$$G_{i8} = -\,\frac{\Sigma_{s_{14}}\,C_{i2} + \Sigma_{s_{24}}\,G_{i2} + \Sigma_{s_{34}}\,G_{i4}}{D_4\,(k_1^2 - k_4^2)}$$

$$G_{i9} = -\,\frac{\Sigma_{s_{24}}\,C_{i3} + \Sigma_{s_{34}}\,G_{i5}}{D_4\,(k_2^2 - k_4^2)}$$

$$G_{i10} = -\,\frac{\Sigma_{s_{24}}\,C_{i4} + \Sigma_{s_{34}}\,G_{i6}}{D_4\,(k_2^2 - k_4^2)}$$

$$G_{i11} = -\,\frac{\Sigma_{s_{34}}\,C_{i5}}{D_4\,(k_3^2 - k_4^2)}$$

$$G_{i12} = -\,\frac{\Sigma_{s_{34}}\,C_{i6}}{D_4\,(k_3^2 - k_4^2)}\quad ,$$

and C_{i7} and C_{i8} are the solutions of the following system

$$- G_{i7}\, e^{-k_1 R}\left[1 + 2\, D_4\, (k_1 + \frac{1}{R}) \right] - G_{i8}\, e^{+k_1 R}\left[1 - 2\, D_4\, (k_1 - \frac{1}{R}) \right]$$

$$- G_{i9}\, e^{-k_2 R}\left[1 + 2\, D_4\, (k_2 + \frac{1}{R}) \right] - G_{i10}\, e^{+k_2 R}\left[1 - 2\, D_4\, (k_2 - \frac{1}{R}) \right]$$

$$- G_{i11}\, e^{-k_3 R}\left[1 + 2\, D_4\, (k_3 + \frac{1}{R}) \right] - G_{i12}\, e^{+k_3 R}\left[1 - 2\, D_4\, (k_3 - \frac{1}{R}) \right] =$$

$$C_{i7}\, e^{-k_4 R}\left[1 + 2\, D_4\, (k_4 + \frac{1}{R}) \right] + C_{i8}\, e^{+k_4 R}\left[1 - 2\, D_4\, (k_4 - \frac{1}{R}) \right]$$

and

$$- G_{i7}\, e^{-k_1 H}\left[1 - 2\, D_4\, (k_1 + \frac{1}{H}) \right] - G_{i8}\, e^{+k_1 H}\left[1 + 2\, D_4\, (k_1 - \frac{1}{H}) \right]$$

$$- G_{i9}\, e^{-k_2 H}\left[1 - 2\, D_4\, (k_2 + \frac{1}{H}) \right] - G_{i10}\, e^{+k_2 H}\left[1 + 2\, D_4\, (k_2 - \frac{1}{H}) \right]$$

$$- G_{i11}\, e^{-k_3 H}\left[1 - 2\, D_4\, (k_3 + \frac{1}{H}) \right] - G_{i12}\, e^{+k_3 H}\left[1 + 2\, D_4\, (k_3 - \frac{1}{H}) \right] =$$

$$C_{i7}\, e^{-k_4 H}\left[1 - 2\, D_4\, (k_4 + \frac{1}{H}) \right] + C_{i8}\, e^{+k_4 H}\left[1 + 2\, D_4\, (k_4 - \frac{1}{H}) \right] .$$

Additionally, the four-group reflector albedo expressions for an infinite reflector are given by

$_i\beta_i$ (i = 1, 2, 3, 4)

$$ {}_i\beta_i = \frac{1 - 2\,D_i\,(k_i + \frac{1}{R})}{1 + 2\,D_i\,(k_i + \frac{1}{R})} \quad . \tag{D-5} $$

$_i\beta_{i+1}$ (i = 1, 2, 3)

$$ {}_i\beta_{i+1} = \frac{1}{2\,R}\left[\,C_{i10}\,e^{-k_{i+1}R} + G_{i13}\,e^{-k_i R}\,\right] \tag{D-6} $$

where

$$ G_{i13} = -\,\frac{\Sigma_{s_{ii+1}}}{D_{i+1}\,(k_i^2 - k_{i+1}^2)}\,C_{i9} $$

$$ C_{i9} = \frac{4\,R\,e^{+k_i R}}{1 + 2\,D_i\,(k_i + \frac{1}{R})} \quad \text{and} $$

$$ C_{i10} = \frac{e^{-k_i R}\left[\,1 + 2\,D_{i+1}\,(k_i + R)\,\right]}{e^{-k_{i+1} R}\left[1 + 2\,D_{i+1}\,(k_{i+1} + R)\,\right]}\,G_{i13} \quad . $$

$\underline{{}_i\beta_{i+2} \;(i = 1, 2)}$

$$ {}_i\beta_{i+2} = \frac{1}{2\,R}\left[\, C_{i11}\, e^{-k_{i+2} R} + G_{i14}\, e^{-k_i R} + G_{i15}\, e^{-k_{i+1} R} \right] \tag{D-7} $$

where

$$ G_{i14} = -\frac{\Sigma_{s_{i\,i+2}}\, C_{i9} + \Sigma_{s_{i+1\,i+2}}\, G_{i13}}{D_{i+2}\,(k_i^2 - k_{i+2}^2)} \;, $$

$$ G_{i15} = -\frac{\Sigma_{s_{i+1\,i+2}}}{D_{i+2}\,(k_{i+1}^2 - k_{i+2}^2)}\, C_{i10} \;, \text{ and} $$

$$ C_{i11} = -\left[e^{-k_i R}\left[1 + 2\,D_{i+2}\,(k_i + \frac{1}{R}) \right] G_{i14} + e^{-k_{i+1} R}\left[1 + 2\,D_{i+2}\,(k_{i+2} + \frac{1}{R}) \right] G_{i15} \right] \Big/ $$
$$ \left[e^{-k_{i+2} R}\left[1 + 2\,D_{i+2}\,(k_{i+2} + \frac{1}{R}) \right]\right] . $$

$\underline{{}_i\beta_4 \;(i = 1)}$

$$ {}_i\beta_4 = \frac{1}{2\,R}\left[\, C_{i12}\, e^{-k_4 R} + G_{i16}\, e^{-k_1 R} + G_{i17}\, e^{-k_2 R} + G_{i18}\, e^{-k_3 R} \right] \tag{D-8} $$

where

$$ G_{i16} = \frac{-\,\Sigma_{s_{14}}\, C_{i9} + \Sigma_{s_{24}}\, G_{i13} + \Sigma_{s_{34}}\, G_{i14}}{D_4\,(k_1^2 - k_4^2)} $$

$$G_{i17} = - \frac{\Sigma_{S_{24}} C_{i10} + \Sigma_{S_{34}} G_{i15}}{D_4 \; (k_2^2 - k_4^2)}$$

$$G_{i18} = - \frac{\Sigma_{S_{34}} C_{i11}}{D_4 \; (k_3^2 - k_4^2)} \quad , \text{ and}$$

$$C_{i12} = - \left[e^{-k_1 R} \left[1 + 2 \, D_4 \, (k_1 + \frac{1}{R}) \right] G_{i16} + e^{-k_2 R} \left[1 + 2 \, D_4 \, (k_2 + \frac{1}{R}) \right] G_{i17} + \right.$$

$$\left. e^{-k_3 R} \left[1 + 2 \, D_4 \, (k_3 + \frac{1}{R}) \right] G_{i18} \right] \Big/ \left[e^{-k_4 R} \left[1 + 2 \, D_4 \, (k_4 + \frac{1}{R}) \right] \right] .$$

Finally, to illustrate the concepts developed in this appendix, three different BeO reflector thicknesses are employed in the albedo calculations. The impact of the reflector thickness on the performance of the reflector albedos can be seen in Table D-1.

TABLE D-1. EFFECTS OF THE BeO REFLECTOR THICKNESS VARIATION ON THE REFLECTOR ALBEDO VALUES USING THE FOUR-GROUP ALBEDO METHOD

${}_{1}\beta_{1'}$	Reflector Thickness in cm		
	100	60	30
${}_{1}\beta_{1}$	.28041	.28041	.28041
${}_{2}\beta_{2}$	.30778	.30778	.30777
${}_{1}\beta_{3}$	.18006	.18006	.11610
${}_{3}\beta_{3}$	.18481	.18006	.14602
${}_{2}\beta_{2}$	.62471	.62471	.62470
${}_{2}\beta_{4}$	.15887	.15887	.15883
${}_{2}\beta_{4}$	.13455	.13202	.11357
${}_{3}\beta_{3}$	.68448	.68448	.68447
${}_{3}\beta_{4}$	.23575	.23344	.21657
${}_{4}\beta_{4}$	.93254	.93111	.92088

- Inner radius of the reflector = R = 70.0 cm.
- Reflector thickness = H - R.
- BeO four-group constants from Table 6-3.

The BeO reflector having a thickness of 100 cm behaves virtually as an infinite reflector. From Table D-1, it is seen that, as the BeO reflector thickness decreases, ${}_i\beta_4$ (i = 1, 2, 3, 4) values decrease quicker than ${}_i\beta_{i'}$ (i' ≠ 4) results. This expected behavior is a direct consequence of the fact that (1) fast neutrons require larger numbers of collisions with BeO nuclei to decrease the neutron energy to the thermal energy range and (2) thermal neutron transmission probability increases as BeO reflector becomes thinner.

The program ALB5 used for these numerical calculations is in Appendix G.

APPENDIX E
FOUR-GROUP P_{o_i} DERIVATION

Here the derivation of the partial principal nonleakage probabilities, P_{o_i} , is derived employing the consistent four-group diffusion theory method. Table 5-2 shows that all fission neutrons are essentially born in the first two energy groups. Thus, it is assumed $\chi_3 = 0$ and $\chi_4 = 0$. The subscript, r, is used to denote the reflector.

Four-Group Diffusion Equations for the Core

The characteristic equations for the core fluxes can be seen as follows

$$- D_1 \nabla^2 \phi_1 + \Sigma_{R_1} \phi_1 = \frac{\chi_1}{k_{eff}} \left[\left(\nu_1 \Sigma_{f_1} + \frac{2 \Sigma_{n,2n_1}}{\chi_1} \right) \phi_1 + \nu_2 \Sigma_{f_2} \phi_2 + \nu_3 \Sigma_{f_3} \phi_3 + \nu_4 \Sigma_{f_4} \phi_4 \right]$$

$$- D_2 \nabla^2 \phi_2 + \Sigma_{R_2} \phi_2 = \Sigma_{s_{12}} \phi_1 + \frac{\chi_2}{k_{eff}} (\nu_1 \Sigma_{f_1} \phi_1 + \nu_2 \Sigma_{f_2} \phi_2 + \nu_3 \Sigma_{f_3} \phi_3 + \nu_4 \Sigma_{f_4} \phi_4)$$

(E-1)

$$- D_3 \nabla^2 \phi_3 + \Sigma_{R_3} \phi_3 = \Sigma_{s_{13}} \phi_1 + \Sigma_{s_{23}} \phi_2$$

$$- D_4 \nabla^2 \phi_4 + \Sigma_{R_4} \phi_4 = \Sigma_{s_{14}} \phi_1 + \Sigma_{s_{24}} \phi_2 + \Sigma_{s_{34}} \phi_3$$

Four-Group Diffusion Equations for the Reflector

The characteristic equations for the reflector fluxes can be written as follows

$$
\begin{aligned}
&- D_{1r} \nabla^2 \phi_{1r} + \Sigma_{R_{1r}} \phi_{1r} = 0 \\
&- D_{2r} \nabla^2 \phi_{2r} + \Sigma_{R_{2r}} \phi_{2r} = \Sigma_{S_{12r}} \phi_{1r} \\
&- D_{3r} \nabla^2 \phi_{3r} + \Sigma_{R_{3r}} \phi_{3r} = \Sigma_{S_{13r}} \phi_{1r} + \Sigma_{S_{23r}} \phi_{2r} \\
&- D_{4r} \nabla^2 \phi_{4r} + \Sigma_{R_{4r}} \phi_{4r} = \Sigma_{S_{14r}} \phi_{1r} + \Sigma_{S_{24r}} \phi_{2r} + \Sigma_{S_{34r}} \phi_{3r}
\end{aligned}
\tag{E-2}
$$

Boundary Conditions for the Core and Reflector Fluxes

The boundary conditions for the core and reflector fluxes are described below

1) at $r = 0$, ϕ_i are finite,

2) at $r = R$, $\phi_i = \phi_{ir}$ and $- D_i \phi_i' = - D_{ir} \phi_i'$, i.e., neutron fluxes and currents must be continuous at the boundary between core and reflector. The prime (′) denotes the gradient of the flux, and

3) at $r = \infty$, ϕ_{ir} are finite.

Core Flux Distributions, ϕ_i ($i = 1, 2, 3, 4$)

Solving the simultaneous linear differential equations with constant coefficients for the core and reflector fluxes and applying the boundary conditions, one can write the core flux solutions

$$\phi_i\ (r) = M_{i1}\ \frac{\sin(\lambda_1\ r)}{r} + M_{i2}\ \frac{\sinh(\lambda_2 r)}{r} + \qquad \text{(E-3)}$$

$$M_{i3}\ \frac{\sinh(\lambda_3\ r)\ \cos(\lambda_4\ r)}{r} + M_{i4}\ \frac{\cosh(\lambda_3\ r)\ \sin(\lambda_4\ r)}{r}\ .$$

The roots λ_1, λ_2, λ_3, λ_4 are obtained from the characteristic equations of the core fluxes. The coefficients M_{i1}, M_{i2}, M_{i3}, and M_{i4} are known from Equation E-1 combined with the boundary condition equations.

Evaluation of P_{o_i}

For a spherical core with an infinite reflector, the ϕ_{o_i} are given by

$$\phi_{o_i}(r) = \phi_i\ (r) - \frac{H_i}{r}\ \sinh\ (L_i\ r)$$

where

$L_i = \sqrt{\frac{\Sigma_{R_i}}{D_i}}$ and subject to the boundary condition

at $r = R$, $J_{o_{i-}} = \frac{\phi_{o_i}}{4} + \frac{D_i}{2}\ \frac{d}{dr}\ \phi_{o_i} = 0$.

After applying the boundary condition above, one can write the expression for H_i as follows

$$H_i = [\, R\, [\, M_{i1} \sin(\lambda_1 R) + M_{i2} \sinh(\lambda_2 R) + M_{i3} \sinh(\lambda_3 R) \cos(\lambda_4 R) +$$

$$M_{i4} \cosh(\lambda_3 R) \sin(\lambda_4 R)\,] + 2 D_i\, [\, M_{i1}\, [\, R\lambda_1 \cos(\lambda_1 R) - \sin(\lambda_1 R)\,] +$$

$$M_{i2}\, [\, R\lambda_2 \cosh(\lambda_2 R) - \sinh(\lambda_2 R)\,] + M_{i3}\, R\, [\, \lambda_3 \cosh(\lambda_3 R) \cos(\lambda_4 R) -$$

$$\lambda_4 \sinh(\lambda_3 R) \sin(\lambda_4 R) - (\sinh(\lambda_3 R) \cos(\lambda_4 R)) \;/\; R\,] +$$

$$M_{i4}\, R\, [\, \lambda_3 \sinh(\lambda_3 R) \sin(\lambda_4 R) + \lambda_4 \cosh(\lambda_3 R) \cos(\lambda_4 R) -$$

$$(\cosh(\lambda_3 R) \sin(\lambda_4 R)) \;/\; R\,]\,] \;/\; [\, R \sinh(L_i R) + 2 D_i\, [\, RL_i \cosh(L_i R) -$$

$$\sinh(L_i R)\,]\,] \quad . \qquad \text{(E-4)}$$

Finally, the initial nonleakage probabilities, P_{o_i} , are given by

$$P_{o_i} = \left[\, 1 + \frac{B_i^2}{L_i^2} \,\right]^{-1}$$

where

$$B_i^2 = -\,\frac{\int_S \nabla \phi_{o_i} \cdot d\vec{S}}{\int_v \phi_{o_i}\, dv} = -\,\frac{\int_{core} \nabla^2 \phi_{o_i}\, dv}{\int_{core} \phi_{o_i}\, dv} =$$

$$= -\,\frac{\int_{r=0}^{R} \frac{d^2}{dr^2} (r\, \phi_{o_i})\, r\, dr}{\int_{r=0}^{R} \phi_{o_i}\, r^2\, dr} \quad . \qquad \text{(E-5)}$$

APPENDIX F
THE PING-PONG DECISION PROCESS FOR A SOURCE
OF GROUP 4 NEUTRONS IN A TWO-REGION
GAS CORE REACTOR PROBLEM

Here is considered the contribution of one group 4 neutron source for evaluation of P_o and P. The neutron fraction transmitted through the core region is δ_4 and the fraction returned as group 4 on this first core boundary traverse is $\delta_4\ {}_4\beta_4$ as can be seen in Figure F-1.

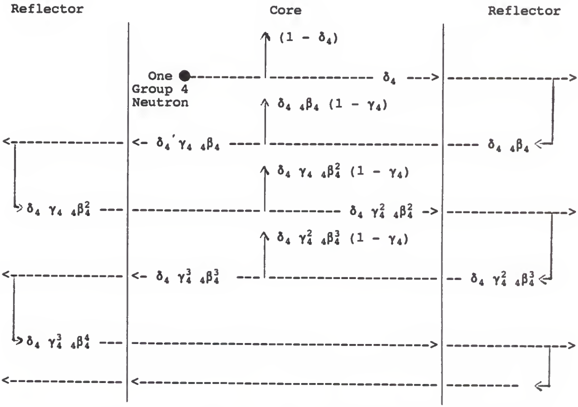


Figure F-1. Schematic drawing showing the ping-pong decision process for a core source of thermal neutrons in a two-region gas core reactor problem.

From Figure F-1, one can infer:

(1) The fraction that interacts into the core is given by

$$H_4 = (1 - \delta_4) + \delta_4 \; {}_4\beta_4 \, (1 - \delta_4) \left[1 + \gamma_4 \; {}_4\beta_4 + (\gamma_4 \; {}_4\beta_4)^2 + \cdots \right] \quad \text{or}$$

$$H_4 = (1 - \delta_4) + \frac{\delta_4 \; {}_4\beta_4 \, (1 - \gamma_4)}{1 - \gamma_4 \; {}_4\beta_4} \; , \tag{F-1}$$

(2) The initial fraction absorbed in the core as group 4 is given by

$$H_4 \times \frac{\Sigma_{a_4}^{core}}{\Sigma_{t_4}^{core}} \; , \tag{F-2}$$

(3) The fraction scattered in the core as group 4 is given by

$$H_4 \times \frac{\Sigma_{s_{44}}^{core}}{\Sigma_{t_4}^{core}} \; . \tag{F-3}$$

Thus, in this first run, one initial group 4 neutron creates another secondary source of group 4 neutron whose strength is given by Equation F-3, and the fraction absorbed in the core as group 4 neutron is given by Equation F-2. Repeating this ping-pong decision process infinite times, i.e., until all secondary sources disappear, the total quantity absorbed as group 4 neutron is given by

$$H_4 \times \frac{\Sigma_{a_4}^{core}}{\Sigma_{t_4}^{core}} \left[1 + H_4 \frac{\Sigma_{s_{44}}^{CORE}}{\Sigma_{t_4}^{CORE}} + \left(H_4 \frac{\Sigma_{s_{44}}^{core}}{\Sigma_{t_4}^{core}} \right)^2 + \cdots \right] \quad \text{or}$$

$$\frac{H_4 \times \dfrac{\Sigma_{a_4}^{core}}{\Sigma_{t_4}^{core}}}{1 - H_4 \times \dfrac{\Sigma_{s_{44}}^{core}}{\Sigma_{t_4}^{core}}} \, . \tag{F-4}$$

Finally, for each group 4 neutron created in the core the quantity given by Equation F-4 is absorbed in the core as group 4 neutron.

APPENDIX G
FOUR-GROUP ALBEDO METHOD COMPUTER PROGRAMS

The four-group albedo approach computer programs used in this work are written in FORTRAN-77 and run on a personal computer. These programs are described below.

Source Listing of ALB1

```
C
C     ***********************************************************
C     FOUR-GROUP ALBEDOS , NONLEAKAGE PROBABILITIES AND EFFECTIVE
C     NEUTRON MULTIPLICATION FACTOR FOR A LARGE OPTICAL-PATH-
C     LENGTH CORE REACTOR (TWO-REGION SOLID CORE REACTOR)
C     ***********************************************************
C
      PROGRAM ALB1
      NTIN=01
      NTOUT=02
      OPEN(NTIN, FILE='ALB1.IN', STATUS='OLD')
      OPEN(NTOUT, FILE='ALB1.OUT', STATUS='UNKNOWN')
C
C     ***********************************************************
C                          INPUT DATA
C
C     R    = RADIUS OF THE SPHERICAL CORE
C     DF   = CORE DENSITY FACTOR
C     EK   = STARTING GUESS FOR THE EFFECTIVE NEUTRON
C            MULTIPLICATION FACTOR
C
C     CORE GROUP PARAMETERS ( SEE TABLE 5-2 )
C
C     D1   = DIFFUSION COEFFICIENT FOR GROUP 1
C     D2   = DIFFUSION COEFFICIENT FOR GROUP 2 , AND SO ON.
C     EN1  = MACROSCOPIC (n,2n) CROSS SECTION FOR GROUP 1
C     EA1  = MACROSCOPIC ABSORPTION CROSS SECTION FOR GROUP 1
C     EA2  = MACROSCOPIC ABSORPTION CROSS SECTION FOR GROUP 2 ,
C            AND SO ON.
C     FV1  = ( AVERAGE NUMBER OF FISSION NEUTRONS RELEASED IN A
C            FISSION REACTION ) * ( MACROSCOPIC FISSION CROSS
C            SECTION ) FOR GROUP 1
C     FV2  = ( AVERAGE NUMBER OF FISSION NEUTRONS RELEASED IN A
C            FISSION REACTION ) * ( MACROSCOPIC FISSION CROSS
C            SECTION ) FOR GROUP 2 , AND SO ON.
C     CHI1 = PROBABILITY THAT A FISSION NEUTRON WILL BE BORN
C            WITH AN ENERGY IN GROUP 1
C     CHI2 = PROBABILITY THAT A FISSION NEUTRON WILL BE BORN
C            WITH AN ENERGY IN GROUP 2
C     ES12 = MACROSCOPIC GROUP-TRANSFER CROSS SECTION FROM
C            GROUP 1 TO GROUP 2
C     ES13 = MACROSCOPIC GROUP-TRANSFER CROSS SECTION FROM
C            GROUP 1 TO GROUP 3 , AND SO ON.
C
C     REFLECTOR GROUP PARAMETERS ( SEE TABLE 5-3 )
C
C     DR1  = DIFFUSION COEFFICIENT FOR GROUP 1
C     DR2  = DIFFUSION COEFFICIENT FOR GROUP 2 , AND SO ON.
C     ERA1 = MACROSCOPIC ABSORPTION CROSS SECTION FOR GROUP 1
C     ERA2 = MACROSCOPIC ABSORPTION CROSS SECTION FOR GROUP 2 ,
C            AND SO ON.
```

```
C     ERS12= MACROSCOPIC GROUP-TRANSFER CROSS SECTION FROM
C            GROUP 1 TO GROUP 2
C     ERS13= MACROSCOPIC GROUP-TRANSFER CROSS SECTION FROM
C            GROUP 1 TO GROUP 3 , AND SO ON.
C
C     ************************************************************
C
      READ(01,1)R,DF,EK
      READ(01,2)D1,D2,D3,D4,EN1,EA1,EA2,EA3,EA4,FV1,FV2,FV3,FV4,
     #ES12,ES13,ES14,ES23,ES24,ES34,CHI1,CHI2,DR1,DR2,DR3,DR4,
     #ERA1,ERA2,ERA3,ERA4,ERS12,ERS13,ERS14,ERS23,ERS24,ERS34
 1    FORMAT(3E17.7)
 2    FORMAT(4E17.7)
      D1=D1/DF
      D2=D2/DF
      D3=D3/DF
      D4=D4/DF
      EA1=EA1*DF
      EA2=EA2*DF
      EA3=EA3*DF
      EA4=EA4*DF
      ES12=ES12*DF
      ES13=ES13*DF
      ES14=ES14*DF
      ES23=ES23*DF
      ES24=ES24*DF
      ES34=ES34*DF
      FV1=FV1*DF
      FV2=FV2*DF
      FV3=FV3*DF
      FV4=FV4*DF
      EN1=EN1*DF
      EA1=EA1+EN1
      ER1=EA1+ES12+ES13+ES14
      ER2=EA2+ES23+ES24
      ER3=EA3+ES34
      ER4=EA4
      ERR1=ERA1+ERS12+ERS13+ERS14
      ERR2=ERA2+ERS23+ERS24
      ERR3=ERA3+ERS34
      ERR4=ERA4
C
C     ***********************************************************
C     PARTIAL PRINCIPAL NONLEAKAGE PROBABILITY CALCULATION
C     ***********************************************************
C
      CK1=SQRT(ER1/D1)
      CK2=SQRT(ER2/D2)
      CK3=SQRT(ER3/D3)
      CK4=SQRT(ER4/D4)
      RK1=SQRT(ERR1/DR1)
      RK2=SQRT(ERR2/DR2)
      RK3=SQRT(ERR3/DR3)
      RK4=SQRT(ERR4/DR4)
      Z1=EXP(-RK1*R)/R
      Z2=EXP(-RK2*R)/R
      Z3=EXP(-RK3*R)/R
      Z4=EXP(-RK4*R)/R
      ZP1=-(R*RK1+1.)*EXP(-RK1*R)/(R*R)
      ZP2=-(R*RK2+1.)*EXP(-RK2*R)/(R*R)
      ZP3=-(R*RK3+1.)*EXP(-RK3*R)/(R*R)
      ZP4=-(R*RK4+1.)*EXP(-RK4*R)/(R*R)
      F1=ERS12/(DR2*(RK2*RK2-RK1*RK1))
      F2=(ERS13+ERS23*F1)/(DR3*(RK3*RK3-RK1*RK1))
      F3=ERS23/(DR3*(RK3*RK3-RK2*RK2))
      F4=(ERS14+ERS24*F1+ERS34*F2)/(DR4*(RK4*RK4-RK1*RK1))
      F5=(ERS24+ERS34*F3)/(DR4*(RK4*RK4-RK2*RK2))
      F6=ERS34/(DR4*(RK4*RK4-RK3*RK3))
      Q11=((CHI1*FV1+2.*EN1)/EK-ER1)/D1
      Q12=CHI1*FV2/(EK*D1)
      Q13=CHI1*FV3/(EK*D1)
      Q14=CHI1*FV4/(EK*D1)
      Q21=(ES12+CHI2*FV1/EK)/D2
      Q22=(CHI2*FV2/EK-ER2)/D2
      Q23=CHI2*FV3/(EK*D2)
      Q24=CHI2*FV4/(EK*D2)
      Q31=ES13/D3
      Q32=ES23/D3
      Q33=-ER3/D3
```

```
      Q41=ES14/D4
      Q42=ES24/D4
      Q43=ES34/D4
      Q44=-ER4/D4
      A1=Q11+Q22+Q33+Q44
      A2=Q11*(Q22+Q33+Q44)+Q22*(Q33+Q44)+Q33*Q44-(Q14*Q41+Q24*
     #Q42+Q13*Q31+Q23*Q32+Q12*Q21)
      A3=-Q14*Q41*(Q22+Q33)+Q14*Q21*Q42+Q14*Q31*Q43+Q24*Q32*Q43+
     #Q24*Q12*Q41-Q24*Q42*(Q11+Q33)+Q44*(Q11*Q22+Q11*Q33+Q22*
     #Q33)-Q44*Q13*Q31-Q44*Q23*Q32-Q44*Q12*Q21+Q11*Q22*Q33+Q12*
     #Q23*Q31+Q21*Q32*Q13-Q13*Q31*Q22-Q23*Q32*Q11-Q12*Q21*Q33
      A4=-Q14*(Q21*Q32*Q43+Q41*Q22*Q33+Q23*Q31*Q42-Q23*Q32*Q41-
     #Q21*Q42*Q33-Q31*Q43*Q22)+Q24*(Q32*Q43*Q11+Q12*Q41*Q33+Q13*
     #Q31*Q42-Q13*Q32*Q41-Q42*Q33*Q11-Q12*Q13*Q43)+Q44*(Q11*Q22*
     #Q33+Q12*Q23*Q31+Q21*Q32*Q13-Q13*Q31*Q22-Q23*Q32*Q11-Q12*
     #Q21*Q33)
      PI=2.*ASIN(1.)
      CA1=-A2
      CA2=A1*A3-4.*A4
      CA3=4.*A2*A4-A3*A3-A1*A1*A4
      QP1=(3.*CA2-CA1*CA1)/9.
      QP2=(9.*CA1*CA2-27.*CA3-2.*CA1**3)/54.
      QP3=QP1**3+QP2*QP2
      QP4=QP2+(ABS(QP3))**0.5
      QP5=QP2-(ABS(QP3))**0.5
      QP6=(ABS(QP4))**(1./3.)
      QP7=(ABS(QP5))**(1./3.)
      IF(QP3)10,52,52
   10 TETA=ACOS(QP2/(-QP1**3)**0.5)
      R1=2.*(-QP1)**0.5*COS(TETA/3.)-CA1/3.
      R2=2.*(-QP1)**0.5*COS(TETA/3.+2.*PI/3.)-CA1/3.
      R3=2.*(-QP1)**0.5*COS(TETA/3.+4.*PI/3.)-CA1/3.
      DSR1=A1*A1-4.*A2+4.*R1
      ESR1=R1*R1-4.*A4
      IF(DSR1)25,20,20
   20 CONTINUE
      IF(ESR1)25,22,22
   22 XR1=R1
      GO TO 64
   25 DSR2=A1*A1-4.*A2+4.*R2
      ESR2=R2*R2-4.*A4
      IF(DSR2)35,30,30
   30 CONTINUE
      IF(ESR2)35,32,32
   32 XR1=R2
      GO TO 64
   35 DSR3=A1*A1-4.*A2+4.*R3
      ESR3=R3*R3-4.*A4
      IF(DSR3)999,40,40
   40 CONTINUE
      IF(ESR3)999,42,42
   42 XR1=R3
      GO TO 64
   52 CONTINUE
      IF(QP4)54,56,56
   54 QP6=-QP6
      GO TO 57
   56 QP6=QP6
   57 CONTINUE
      IF(QP5)58,60,60
   58 QP7=-QP7
      GO TO 61
   60 QP7=QP7
   61 CONTINUE
      XR1=QP6+QP7-CA1/3.
      XRR=-(QP6+QP7)/2.-CA1/3.
      XIR=(QP6-QP7)*COS(PI/6.)
   64 DIS=A1*A1-4.*A2+4.*XR1
      EIS=XR1*XR1-4.*A4
      IF(DIS)999,66,66
   66 CONTINUE
      IF(EIS)999,67,67
   67 CONTINUE
      QP8=(A1-SQRT(DIS))/2.
      QP9=(XR1+SQRT(EIS))/2.
      QP10=(A1+SQRT(DIS))/2.
      QP11=(XR1-SQRT(EIS))/2.
      DIS1=QP8*QP8-4.*QP9
      QP12=(ABS(DIS1))**0.5
```

```
      IF(DIS1)74,70,70
70    ZZ1=(-QP8+QP12)/2.
      ZZ2=(-QP8-QP12)/2.
      GO TO 81
74    RZZ1=-QP8/2.
      RIZ1=QP12/2.
      TETA1=ATAN(ABS(RIZ1/RZZ1))
      IF(RZZ1)77,78,78
77    AA=(RZZ1*RZZ1+RIZ1*RIZ1)**0.25*COS((PI-TETA1)/2.)
      BB=(RZZ1*RZZ1+RIZ1*RIZ1)**0.25*SIN((PI-TETA1)/2.)
      A=AA
      B=BB
      GO TO 81
78    AA=(RZZ1*RZZ1+RIZ1*RIZ1)**0.25*COS(TETA1/2.)
      BB=(RZZ1*RZZ1+RIZ1*RIZ1)**0.25*SIN(TETA1/2.)
      A=AA
      B=BB
81    DIS2=QP10*QP10-4.*QP11
      QP13=(ABS(DIS2))**0.5
      IF(DIS2)86,82,82
82    ZZ3=(-QP10+QP13)/2.
      ZZ4=(-QP10-QP13)/2.
      ZZ1=ZZ3
      ZZ2=ZZ4
      GO TO 99
86    RZZ3=-QP10/2.
      RIZ3=QP13/2.
      TETA2=ATAN(ABS(RIZ3/RZZ3))
      IF(RZZ3)90,91,91
90    A=(RZZ3*RZZ3+RIZ3*RIZ3)**0.25*COS((PI-TETA2)/2.)
      B=(RZZ3*RZZ3+RIZ3*RIZ3)**0.25*SIN((PI-TETA2)/2.)
      GO TO 99
91    A=(RZZ3*RZZ3+RIZ3*RIZ3)**0.25*COS(TETA2/2.)
      B=(RZZ3*RZZ3+RIZ3*RIZ3)**0.25*SIN(TETA2/2.)
99    CONTINUE
      SLAMB=ZZ1
      SMU=-ZZ2
      SU=SQRT(SMU)
      SL=SQRT(SLAMB)
      X1=SIN(SU*R)/R
      X2=SINH(SL*R)/R
      X3=SINH(A*R)*COS(B*R)/R
      X4=COSH(A*R)*SIN(B*R)/R
      XP1=(R*SU*COS(SU*R)-SIN(SU*R))/(R*R)
      XP2=(R*SL*COSH(SL*R)-SINH(SL*R))/(R*R)
      XP3=(R*A*COSH(A*R)*COS(B*R)-R*B*SINH(A*R)*SIN(B*R)-
     #SINH(A*R)*COS(B*R))/(R*R)
      XP4=(R*A*SINH(A*R)*SIN(B*R)+R*B*COSH(A*R)*COS(B*R)-
     #COSH(A*R)*SIN(B*R))/(R*R)
      Q34=Q24*(Q43*Q32-Q42*(Q33-SMU))-(Q44-SMU)*(Q32*Q23-(Q22-
     #SMU)*(Q33-SMU))
      Q50=(Q24*(Q31*Q42-Q32*Q41)-(Q44-SMU)*(Q31*(Q22-SMU)-
     #Q32*Q21))/Q34
      Q51=(-Q31-(Q33-SMU)*Q50)/Q32
      Q52=(-Q41-Q43*Q50-Q42*Q51)/(Q44-SMU)
      Q35=Q24*(Q32*Q43-Q42*(Q33+SL*SL))+(Q44+SL*SL)*((Q22+SL*SL)
     #*(Q33+SL*SL)-Q23*Q32)
      Q53=(Q24*(Q31*Q42-Q32*Q41)+(Q44+SL*SL)*(Q21*Q32-Q31*(Q22+
     #SL*SL)))/Q35
      Q54=(-Q31-(Q33+SL*SL)*Q53)/Q32
      Q55=(-Q41-Q43*Q53-Q42*Q54)/(Q44+SL*SL)
      Q60=2.*A*B
      Q61=Q11+A*A-B*B
      Q62=Q33+A*A-B*B
      Q63=Q44+A*A-B*B
      Q64=Q42-Q63*Q12/Q14
      Q65=-Q60*Q12/Q14
      Q66=Q43-Q63*Q13/Q14
      Q67=-Q60*Q13/Q14
      Q68=-Q63*Q61/Q14+Q60*Q60/Q14+Q41
      Q69=-Q63*Q60/Q14-Q60*Q61/Q14
      Q70=Q60*Q12/Q14
      Q71=Q42-Q63*Q12/Q14
      Q72=Q60*Q13/Q14
      Q73=Q43-Q63*Q13/Q14
      Q75=Q60*(Q61+Q63)/Q14
      Q76=Q60*Q60/Q14-Q63*Q61/Q14+Q41
      Q77=-Q64*Q62/Q32+Q65*Q60/Q32+Q66
      Q78=-Q64*Q60/Q32-Q65*Q62/Q32+Q67
```

```
Q79=-Q64*Q31/Q32+Q68
Q80=-Q65*Q31/Q32+Q69
Q81=-Q70*Q62/Q32+Q71*Q60/Q32+Q72
Q82=-Q70*Q60/Q32-Q71*Q62/Q32+Q73
Q83=-Q70*Q31/Q32+Q75
Q84=-Q71*Q31/Q32+Q76
Q36=Q77*Q82-Q78*Q81
Q85=(Q78*Q83-Q79*Q82)/Q36
Q86=(Q78*Q84-Q80*Q82)/Q36
Q87=(Q79*Q81-Q77*Q83)/Q36
Q88=(Q80*Q81-Q77*Q84)/Q36
Q89=-(Q31+Q62*Q85+Q60*Q87)/Q32
Q90=-(Q62*Q86+Q60*Q88)/Q32
Q91=(Q60*Q85-Q62*Q87)/Q32
Q92=(-Q31+Q60*Q86-Q62*Q88)/Q32
Q93=-(Q61+Q12*Q89+Q13*Q85)/Q14
Q94=-(Q60+Q12*Q90+Q13*Q86)/Q14
Q95=(Q60-Q12*Q91-Q13*Q87)/Q14
Q96=-(Q61+Q12*Q92+Q13*Q88)/Q14
Q26=A*A-B*B+Q22
Q100=-Q26*Q43/Q42+Q23
Q101=-Q60*Q43/Q42
Q102=-Q26*Q63/Q42+Q24+Q60*Q60/Q42
Q103=-(Q26+Q63)*Q60/Q42
Q104=Q26*Q41/Q42-Q21
Q105=Q60*Q41/Q42
Q106=Q60*Q43/Q42
Q107=-Q26*Q43/Q42+Q23
Q108=(Q63+Q26)*Q60/Q42
Q109=Q60*Q60/Q42-Q26*Q63/Q42+Q24
Q110=-Q60*Q41/Q42
Q111=Q26*Q41/Q42-Q21
Q112=-Q62/Q32+Q43/Q42
Q113=-Q60/Q32
Q114=Q63/Q42
Q115=Q60/Q42
Q116=-Q41/Q42+Q31/Q32
Q117=Q60/Q32
Q118=-Q62/Q32+Q43/Q42
Q119=-Q60/Q42
Q120=Q63/Q42
Q121=-Q41/Q42+Q31/Q32
Q122=Q112-Q115*Q117/Q120
Q123=Q113-Q115*Q118/Q120
Q124=Q114-Q115*Q119/Q120
Q125=-Q115*Q121/Q120
Q126=Q106-Q108*Q122/Q124-Q109*Q117/Q120
Q127=Q107-Q108*Q123/Q124-Q109*Q118/Q120
Q128=-Q109*Q119/Q120
Q129=Q110-Q108*Q116/Q124
Q130=Q111-Q108*Q125/Q124-Q109*Q121/Q120
Q131=Q126-Q128*Q122/Q124
Q132=Q127-Q128*Q123/Q124
Q133=Q129-Q128*Q116/Q124
Q134=Q130-Q128*Q125/Q124
Q135=Q100-Q102*Q122/Q124-Q103*Q117/Q120
Q136=Q101-Q102*Q123/Q124-Q103*Q118/Q120
Q137=-Q103*Q119/Q120
Q138=Q104-Q102*Q116/Q124
Q139=Q105-Q103*Q121/Q120-Q102*Q125/Q124
Q140=Q135-Q137*Q122/Q124
Q141=Q136-Q137*Q123/Q124
Q142=Q138-Q137*Q116/Q124
Q143=Q139-Q137*Q125/Q124
Q144=Q131*Q141-Q132*Q140
Q145=(Q141*Q133-Q132*Q142)/Q144
Q146=(Q141*Q134-Q132*Q143)/Q144
Q147=(Q131*Q142-Q133*Q140)/Q144
Q148=(Q131*Q143-Q134*Q140)/Q144
Q149=-(Q31+Q62*Q145+Q60*Q147)/Q32
Q150=-(Q62*Q146+Q60*Q148)/Q32
Q151=(Q60*Q145-Q62*Q147)/Q32
Q152=(Q60*Q146-Q31-Q62*Q148)/Q32
Q153=(Q116-Q122*Q145-Q123*Q147)/Q124
Q154=(Q125-Q122*Q146-Q123*Q148)/Q124
Q155=-(Q117*Q145+Q118*Q147+Q119*Q153)/Q120
Q156=(Q121-Q117*Q146-Q118*Q148-Q119*Q154)/Q120
S11=Q51
S22=Q54
```

```
S33=Q149
S34=Q150
S43=Q151
S44=Q152
AM11=Q50
AM22=Q53
AM33=Q145
AM34=Q146
AM43=Q147
AM44=Q148
AN11=Q52
AN22=Q55
AN33=Q153
AN34=Q154
AN43=Q155
AN44=Q156
Y11=X1
Y12=X2
Y13=X3
Y14=X4
Y15=Z1
Y21=S11*X1
Y22=S22*X2
Y23=S33*X3+S43*X4
Y24=S34*X3+S44*X4
Y25=F1*Z1
Y26=Z2
Y31=AM11*X1
Y32=AM22*X2
Y33=AM33*X3+AM43*X4
Y34=AM34*X3+AM44*X4
Y35=F2*Z1
Y36=F3*Z2
Y37=Z3
Y41=AN11*X1
Y42=AN22*X2
Y43=AN33*X3+AN43*X4
Y44=AN34*X3+AN44*X4
Y45=F4*Z1
Y46=F5*Z2
Y47=F6*Z3
Y48=Z4
Y51=XP1
Y52=XP2
Y53=XP3
Y54=XP4
Y55=DR1*ZP1/D1
Y61=S11*XP1
Y62=S22*XP2
Y63=S33*XP3+S43*XP4
Y64=S34*XP3+S44*XP4
Y65=DR2*F1*ZP1/D2
Y66=DR2*ZP2/D2
Y71=AM11*XP1
Y72=AM22*XP2
Y73=AM33*XP3+AM43*XP4
Y74=AM34*XP3+AM44*XP4
Y75=DR3*F2*ZP1/D3
Y76=DR3*F3*ZP2/D3
Y77=DR3*ZP3/D3
X60=Y31-Y35*Y11/Y15-Y36*(Y21-Y25*Y11/Y15)/Y26
X61=Y32-Y35*Y12/Y15-Y36*(Y22-Y25*Y12/Y15)/Y26
X62=Y33-Y35*Y13/Y15-Y36*(Y23-Y25*Y13/Y15)/Y26
X63=Y34-Y35*Y14/Y15-Y36*(Y24-Y25*Y14/Y15)/Y26
X65=Y71-Y75*Y11/Y15-Y76*(Y21-Y25*Y11/Y15)/Y26
X66=Y72-Y75*Y12/Y15-Y76*(Y22-Y25*Y12/Y15)/Y26
X67=Y73-Y75*Y13/Y15-Y76*(Y23-Y25*Y13/Y15)/Y26
X68=Y74-Y75*Y14/Y15-Y76*(Y24-Y25*Y14/Y15)/Y26
X50=Y11*Y55-Y15*Y51
X51=Y12*Y55-Y15*Y52
X52=Y13*Y55-Y15*Y53
X53=Y14*Y55-Y15*Y54
X55=Y66*(Y21*Y15-Y25*Y11)-Y26*(Y61*Y15-Y65*Y11)
X56=Y66*(Y22*Y15-Y25*Y12)-Y26*(Y62*Y15-Y65*Y12)
X57=Y66*(Y23*Y15-Y25*Y13)-Y26*(Y63*Y15-Y65*Y13)
X58=Y66*(Y24*Y15-Y25*Y14)-Y26*(Y64*Y15-Y65*Y14)
X70=Y77*X60-Y37*X65
X71=Y77*X61-Y37*X66
X72=Y77*X62-Y37*X67
```

```
      X73=Y77*X63-Y37*X68
      DELT=X51*(X57*X73-X58*X72)-X52*(X56*X73-X58*X71)+X53*(X56*
     #X72-X57*X71)
      DELT2=-X50*(X57*X73-X58*X72)-X52*(X58*X70-X55*X73)+X53*
     #(X57*X70-X55*X72)
      DELT3=X51*(X58*X70-X55*X73)+X50*(X56*X73-X58*X71)+X53*(X55*
     #X71-X56*X70)
      DELT4=X51*(X55*X72-X57*X70)-X52*(X55*X71-X56*X70)-X50*(X56*
     #X72-X57*X71)
      Q2R=DELT2/DELT
      Q3R=DELT3/DELT
      Q4R=DELT4/DELT
      V=A*A+B*B
      V1=A*A-B*B
      V2=2.*A*B
      V3=SINH(A*R)
      V4=COSH(A*R)
      V5=SIN(B*R)
      V6=COS(B*R)
      V7=SINH(CK1*R)
      V8=COSH(CK1*R)
      V9=SINH(CK2*R)
      V10=COSH(CK2*R)
      V11=SINH(CK3*R)
      V12=COSH(CK3*R)
      V13=SINH(CK4*R)
      V14=COSH(CK4*R)
      AA1=V7/R+2.*D1*(R*CK1*V8-V7)/(R*R)
      A1R=(X1+2.*D1*XP1+Q2R*(X2+2.*D1*XP2)+Q3R*(X3+2.*D1*XP3)+
     #Q4R*(X4+2.*D1*XP4))/AA1
      W1=SIN(SU*R)/(SU*SU)-R*COS(SU*R)/SU
      W2=R*COSH(SL*R)/SL-SINH(SL*R)/(SL*SL)
      W3=(R*A*V4*V6+R*B*V3*V5)/V-(A*A*V3*V6+A*B*V4*V5)/V**2-
     #(B*A*V4*V5-B*B*V3*V6)/V**2
      W4=(R*A*V3*V5-R*B*V4*V6)/V-(A*A*V4*V5-R*B*V3*V6)/V**2+
     #(B*A*V3*V6+B*B*V4*V5)/V**2
      WN1=R*V8/CK1-V7/(CK1*CK1)
      BUC1=-(-SU*SU*W1+Q2R*SL*SL*W2+Q3R*(V1*W3-V2*W4)+Q4R*(
     #V1*W4+V2*W3)-A1R*CK1*CK1*WN1)/(W1+Q2R*W2+Q3R*W3+Q4R*W4-
     #A1R*WN1)
      PO1=1./(1.+BUC1/(CK1*CK1))
      AA2=V9/R+2.*D2*(R*CK2*V10-V9)/(R*R)
      A2R=(S11*(X1+2.*D2*XP1)+S22*Q2R*(X2+2.*D2*XP2)+(S33*Q3R+
     #S34*Q4R)*(X3+2.*D2*XP3)+(S43*Q3R+S44*Q4R)*(X4+2.*D2*XP4))
     #/AA2
      WN2=R*V10/CK2-V9/(CK2*CK2)
      BUC2=-(-S11*SU*SU*W1+S22*Q2R*SL*SL*W2+(S33*Q3R+S34*Q4R)*(
     #V1*W3-2.*A*B*W4)+(S43*Q3R+S44*Q4R)*(V1*W4+2.*A*B*W3)-A2R*
     #CK2*CK2*WN2)/(S11*W1+S22*Q2R*W2+(S33*Q3R+S34*Q4R)*W3+(S43*
     #Q3R+S44*Q4R)*W4-A2R*WN2)
      PO2=1./(1.+BUC2/(CK2*CK2))
      AA3=V11/R+2.*D3*(R*CK3*V12-V11)/(R*R)
      A3R=(AM11*(X1+2.*D3*XP1)+AM22*Q2R*(X2+2.*D3*XP2)+(AM33*
     #Q3R+AM34*Q4R)*(X3+2.*D3*XP3)+(AM43*Q3R+AM44*Q4R)*(X4+2.*
     #D3*XP4))/AA3
      WN3=R*V12/CK3-V11/(CK3*CK3)
      BUC3=-(-AM11*SU*SU*W1+AM22*Q2R*SL*SL*W2+(AM33*Q3R+AM34*
     #Q4R)*(V1*W3-2.*A*B*W4)+(AM43*Q3R+AM44*Q4R)*(V1*W4+V2*W3)-
     #A3R*CK3*CK3*WN3)/(AM11*W1+AM22*Q2R*W2+(AM33*Q3R+AM34*Q4R)*
     #W3+(AM43*Q3R+AM44*Q4R)*W4-A3R*WN3)
      PO3=1./(1.+BUC3/(CK3*CK3))
      AA4=V13/R+2.*D4*(R*CK4*V14-V13)/(R*R)
      A4R=(AN11*(X1+2.*D4*XP1)+AN22*Q2R*(X2+2.*D4*XP2)+(AN33*
     #Q3R+AN34*Q4R)*(X3+2.*D4*XP3)+(AN43*Q3R+AN44*Q4R)*(X4+2.*
     #D4*XP4))/AA4
      WN4=R*V14/CK4-V13/(CK4*CK4)
      BUC4=-(-AN11*SU*SU*W1+AN22*Q2R*SL*SL*W2+(AN33*Q3R+AN34*
     #Q4R)*(V1*W3-V2*W4)+(AN43*Q3R+AN44*Q4R)*(V1*W4+V2*W3)-
     #A4R*CK4*CK4*WN4)/(AN11*W1+AN22*Q2R+(AN33*Q3R+AN34*Q4R)*
     #W3+(AN43*Q3R+AN44*Q4R)*W4-A4R*WN4)
      PO4=1./(1.+BUC4/(CK4*CK4))
C
C     **********************************************************
C     CORE ALBEDO CALCULATION
C     **********************************************************
C
      G11=CK1/TANH(CK1*R)-1./R
      G12=CK2/TANH(CK2*R)-1./R
      G13=CK3/TANH(CK3*R)-1./R
```

```
      G14=CK4/TANH(CK4*R)-1./R
      CA11=(1.-2.*D1*G11)/(1.+2.*D1*G11)
      CA22=(1.-2.*D2*G12)/(1.+2.*D2*G12)
      CA33=(1.-2.*D3*G13)/(1.+2.*D3*G13)
      CA44=(1.-2.*D4*G14)/(1.+2.*D4*G14)
      G15=4.*R/((1.+2.+D1*G11)*SINH(CK1*R))
      G16=G15*ES12/(D2*(CK2*CK2-CK1*CK1))
      G17=-G15*SINH(CK1*R)*(1.+2.*D2*G11)/(SINH(CK2*R)*(1.+2.*
     #D2*G12))
      G18=(G15*ES13+G16*ES23)/(D3*(CK3*CK3-CK1*CK1))
      G19=(G17*ES23)/(D3*(CK3*CK3-CK2*CK2))
      G20=-(G18*SINH(CK1*R)*(1.+2.*D3*G11)+G19*SINH(CK2*R)*(1.+
     #2.*D3*G12))/(SINH(CK3*R)*(1.+2.*D3*G13))
      G21=(G15*ES14+G16*ES24+G18*ES34)/(D4*(CK4*CK4-CK1*CK1))
      G22=(G17*ES24+G19*ES34)/(D4*(CK4*CK4-CK1*CK1))
      G23=G20*ES34/(D4*(CK4*CK4-CK1*CK1))
      G24=-(G21*V7*(1.+2.*D4*G11)+G22*V9*(1.+2.*D4*
     #G12)+G23*V11*(1.+2.*D4*G13))/(V13*(1.+2.*D4*G14))
      CA12=(G17*SINH(CK2*R)+G16*SINH(CK1*R))/(2.*R)
      CA13=(G20*V11+G18*V7+G19*V9)/(2.*R)
      CA14=(G24*V13+G21*V7+G22*V9+G23*V11)/(2.*R)
      G25=4.*R/((1.+2.*D2*G12)*SINH(CK2*R))
      G26=G25*ES23/(D3*(CK3*CK3-CK2*CK2))
      G27=-(G26*V9*(1.+2.*D3*G12))/(V11*(1.+2.*D3*G13))
      G28=(G25*ES24+G26*ES34)/(D4*(CK4*CK4-CK2*CK2))
      G29=G27*ES34/(D4*(CK4*CK4-CK3*CK3))
      G30=-(G28*V9*(1.+2.*D4*G12)+G29*V11*(1.+2.*D4*G13))/
     #(V13*(1.+2.*D4*G14))
      CA23=(G27*SINH(CK3*R)+G26*SINH(CK2*R))/(2.*R)
      CA24=(G30*V13+G28*V9+G29*V11)/(2.*R)
      G31=4.*R/((1.+2.*D3*G13)*SINH(CK3*R))
      G32=G31*ES34/(D4*(CK4*CK4-CK3*CK3))
      G33=-G32*V11*(1.+2.*D4*G13)/(V13*(1.+2.*D4*G14))
      CA34=(G33*V13+G32*V11)/(2.*R)
C
C     ************************************************************
C     REFLECTOR ALBEDO CALCULATION
C     ************************************************************
C
      G34=RK1+1./R
      G35=RK2+1./R
      G36=RK3+1./R
      G37=RK4+1./R
      RA11=(1.-2.*DR1*G34)/(1.+2.*DR1*G34)
      RA22=(1.-2.*DR2*G35)/(1.+2.*DR2*G35)
      RA33=(1.-2.*DR3*G36)/(1.+2.*DR3*G36)
      RA44=(1.-2.*DR4*G37)/(1.+2.*DR4*G37)
      G38=4./(Z1*(1.+2.*DR1*G34))
      G39=G38*ERS12/(DR2*(RK2*RK2-RK1*RK1))
      G40=-G39*Z1*(1.+2.*DR2*G34)/(Z2*(1.+2.*DR2*G35))
      G41=(G38*ERS13+G39*ERS23)/(DR3*(RK3*RK3-RK1*RK1))
      G42=G40*ERS23/(DR3*(RK3*RK3-RK2*RK2))
      G43=-(G41*Z1*(1.+2.*DR3*G34)+G42*Z2*(1.+2.*DR3*G35))/(Z3*(
     #1.+2.*DR3*G36))
      G44=(G38*ERS14+G39*ERS24+G41*ERS34)/(DR4*(RK4*RK4-RK1*RK1))
      G45=(G40*ERS24+G42*ERS34)/(DR4*(RK4*RK4-RK2*RK2))
      G46=G43*ERS34/(DR4*(RK4*RK4-RK3*RK3))
      G47=-(G44*Z1*(1.+2.*DR4*G34)+G45*Z2*(1.+2.*DR4*G35)+G46*Z3
     #*(1.+2.*DR4*G36))/(Z4*(1.+2.*DR4*G37))
      RA12=(G40*Z2+G39*Z1)/2.
      RA13=(G43*Z3+G41*Z1+G42*Z2)/2.
      RA14=(G47*Z4+G44*Z1+G45*Z2+G46*Z3)/2.
      G48=4./(Z2*(1.+2.*DR2*G35))
      G49=G48*ERS23/(DR3*(RK3*RK3-RK2*RK2))
      G50=-G49*Z2*(1.+2.*DR3*G35)/(Z3*(1.+2.*DR3*G36))
      G51=(G48*ERS24+G49*ERS34)/(DR4*(RK4*RK4-RK2*RK2))
      G52=G50*ERS34/(DR4*(RK4*RK4-RK3*RK3))
      G53=-(G51*Z2*(1.+2.*DR4*G35)+G52*Z3*(1.+2.*DR4*G36))/(Z4
     #*(1.+2.*DR4*G37))
      RA23=(G50*Z3+G49*Z2)/2.
      RA24=(G53*Z4+G51*Z2+G52*Z3)/2.
      G54=4./(Z3*(1.+2.*DR3*G36))
      G55=G54*ERS34/(DR4*(RK4*RK4-RK3*RK3))
      G56=-G55*Z3*(1.+2.*DR4*G36)/(Z4*(1.+2.*DR4*G37))
      RA34=(G56*Z4+G55*Z3)/2.
C
C     ************************************************************
C     SOURCE OF INITIAL LEAKAGE CALCULATION
C     ************************************************************
```

```
C
      SI1=CHI1*(1.-PO1)
      S61=CHI2+CHI1*PO1*ES12/ER1
      S62=CHI1*PO1*ES13/ER1+S61*PO2*ES23/ER2
      S63=CHI1*PO1*ES14/ER1+S61*PO2*ES24/ER2+S62*PO3*ES34/ER3
      SI2=S61*(1.-PO2)
      SI3=S62*(1.-PO3)
      SI4=S63*(1.-PO4)
C
C     **********************************************************
C     FRACTIONAL SECONDARY NONLEAKAGE PROBABILITY CALCULATION
C     **********************************************************
C
      G57=1.-CA11*RA11
      G58=1.-CA22*RA22
      G59=1.-CA33*RA33
      G60=1.-CA44*RA44
      PR44=RA44*(1.-CA44)/G60
      P4=SI4*PR44
      PR34=RA34*(1.-CA44)/(G59*G60)
      PR33=RA33*(1.-CA33)/G59-RA33*CA34*(1.-RA44)/(G59*G60)
      P3=SI3*(PR33+PR34)
      PR24=RA24*(1.-CA44)/(G58*G60)
      G61=(1.-CA33)/G59+CA33*RA34*(1.-CA44)/(G59*G60)
      G62=CA34*(1.-RA44)/(G59*G60)
      PR23=RA23*(G61-G62)/G58
      G63=RA22*(1.-CA22)/G58
      G64=RA22*CA23*(1.-P3/SI3)/G58
      G65=RA22*CA24*(1.-PR44)/G58
      PR22=G63-G64-G65
      P2=SI2*(PR24+PR23+PR22)
      PR14=RA14*(1.-CA44)/(G57*G60)
      PR13=RA13*(G61-G62)/G57
      G66=(1.-CA22)*G59*G60-CA24*(1.-RA44)*G59+CA22*RA24*(1.-
     #CA44)*G59
      G67=CA23*((1.-RA33)*G60+RA33*CA34*(1.-RA44)-RA34*(1.-CA44))
      G68=CA22*RA23*((1.-CA33)*G60+CA33*RA34*(1.-CA44)-CA34*(1.-
     #RA44))
      PR12=RA12*(G66-G67+G68)/(G57*G58*G59*G60)
      PR11=RA11*(1.-CA11)/G57-RA11*CA14*(1.-P4/SI4)/G57-RA11*
     #CA13*(1.-P3/SI3)/G57-RA11*CA12*(1.-P2/SI2)/G57
      P1=SI1*(PR14+PR13+PR12+PR11)
      PNP=1.-(SI1+SI2+SI3+SI4)
      SNP=P1+P2+P3+P4
      TNP=PNP+SNP
C
C     **********************************************************
C     CALCULATION OF THE NEUTRON ABSORPTION BY THE CORE
C     **********************************************************
C
      POB1=CHI1*PO1*EA1/ER1
      POB2=S61*PO2*EA2/ER2
      POB3=S62*PO3*EA3/ER3
      POB4=S63*PO4
      BPO=POB1+POB2+POB3+POB4
      PA34=(RA34*PR44/RA44+RA33*CA34*PR44)/G59
      PA33=RA33*(1.-CA34-CA33)/G59
      TA34=(CA34*PR44+CA33*RA34*PR44/RA44)/G59
      TA33=(1.-CA34-CA33)/G59
      PA24=(RA22*CA24*PR44+RA24*PR44/RA44+RA22*CA23*PA34+
     #RA23*TA34)/G58
      PA23=(RA22*CA23*PA33+RA23*TA33)/G58
      PA22=RA22*(1.-CA22-CA23-CA24)/G58
      TA24=(CA24*PR44+CA22*RA24*PR44/RA44+CA23*PA34+CA22*
     #RA23*TA34)/G58
      TA23=(CA23*PA33+CA22*RA23*TA33)/G58
      TA22=(1.-CA22-CA23-CA24)/G58
      PA14=(RA11*CA14*PR44+RA14*PR44/RA44+RA11*CA13*PA34+
     #RA13*TA34+RA11*CA12*PA24+RA12*TA24)/G57
      PA13=(RA11*CA13*PA33+RA13*TA33+RA11*CA12*PA23+
     #RA12*TA23)/G57
      PA12=(RA11*CA12*PA22+RA12*TA22)/G57
      PA11=RA11*(1.-CA11-CA12-CA13-CA14)/G57
      FF34=ES34/ER3
      FF33=EA3/ER3
      FF24=ES24/ER2+ES23*ES34/(ER2*ER3)
      FF23=ES23*EA3/(ER2*ER3)
      FF22=EA2/ER2
      FF14=ES14/ER1+ES12*(ES24+ES23*ES34/ER3)/(ER1*ER2)+
```

```
     #ES13*ES34/(ER1*ER3)
      FF13=ES13*EA3/(ER1*ER3)+ES12*ES23*EA3/(ER1*ER2*ER3)
      FF12=ES12*EA2/(ER1*ER2)
      FF11=EA1/ER1
      E1A1=PA11*FF11
      E1A2=PA11*FF12+PA12*FF22
      E1A3=PA11*FF13+PA12*FF23+PA13*FF33
      E1A4=PA11*FF14+PA12*FF24+PA13*FF34+PA14
      E2A2=PA22*FF22
      E2A3=PA22*FF23+PA23*FF33
      E2A4=PA22*FF24+PA23*FF34+PA24
      E3A3=PA33*FF33
      E3A4=PA33*FF34+PA34
      E4A4=PR44
      PPB1=SI1*E1A1
      PPB2=SI1*E1A2+SI2*E2A2
      PPB3=SI1*E1A3+SI2*E2A3+SI3*E3A3
      PPB4=SI1*E1A4+SI2*E2A4+SI3*E3A4+SI4*E4A4
      BPP=PPB1+PPB2+PPB3+PPB4
      TAB1=POB1+PPB1
      TAB2=POB2+PPB2
      TAB3=POB3+PPB3
      TAB4=POB4+PPB4
      BTNL=TAB1+TAB2+TAB3+TAB4
C
C     **********************************************************
C     EFFECTIVE NEUTRON MULTIPLICATION FACTOR CALCULATION
C     **********************************************************
C
      EKEF=TAB1*(FV1+2.*EN1)/EA1+TAB2*FV2/EA2+TAB3*FV3/EA3+TAB4*
     #FV4/EA4
C
C     **********************************************************
C                   OUTPUT RESULTS
C
C     CORE ALBEDOS ( SEE TABLE 5-4 )
C
C     CA11 = PROBABILITY THAT A NEUTRON FROM GROUP 1 BE REFLECTED
C            WITH GROUP 1 ENERGY
C     CA12 = PROBABILITY THAT A NEUTRON FROM GROUP 1 BE REFLECTED
C            WITH GROUP 2 ENERGY , AND SO ON.
C
C     REFLECTOR ALBEDOS ( SEE TABLE 5-5 )
C
C     RA11 = PROBABILITY THAT A NEUTRON FROM GROUP 1 BE REFLECTED
C            WITH GROUP 1 ENERGY
C     RA12 = PROBABILITY THAT A NEUTRON FROM GROUP 1 BE REFLECTED
C            WITH GROUP 2 ENERGY , AND SO ON.
C
C     PARTIAL PRINCIPAL NONLEAKAGE PROBABILITIES ( SEE
C     TABLE 5-6 )
C
C     PO1  = PROBABILITY THAT GROUP 1 NEUTRONS NEVER SPEND TIME
C            IN THE REFLECTOR
C     PO2  = PROBABILITY THAT GROUP 2 NEUTRONS NEVER SPEND TIME
C            IN THE REFLECTOR , AND SO ON.
C
C     SOURCES OF INITIAL LEAKAGE ( SEE TABLE 5-6 )
C
C     SI1  = FRACTION OF GROUP 1 NEUTRONS THAT LEAKS FROM THE
C            CORE FOR THE FIRST TIME
C     SI2  = FRACTION OF GROUP 2 NEUTRONS THAT LEAKS FROM THE
C            CORE FOR THE FIRST TIME , AND SO ON.
C
C     FRACTIONAL SECONDARY NONLEAKAGE PROBABILITIES ( SEE
C     TABLE 5-7 )
C
C     PR11 = PROBABILITY THAT IF ONE NEUTRON LEAKS AS A GROUP
C            1 NEUTRON FROM THE CORE , AND RETURNS AS A GROUP
C            1 NEUTRON , IT WILL BE ABSORBED IN THE CORE
C     PR12 = PROBABILITY THAH IF ONE NEUTRON LEAKS AS A GROUP
C            1 NEUTRON FROM THE CORE , AND RETURNS AS A GROUP
C            2 NEUTRON , IT WILL BE ABSORBED IN THE CORE ,
C           AND SO ON.
C
C     NEUTRON ABSORPTION ( SEE TABLE 5-8 )
C
C     POB1 = FRACTION OF FISSION NEUTRONS ( NEVER SPEND TIME
C            IN THE REFLECTOR ) WHICH ARE ABSORBED BY THE
```

```
C           CORE AS GROUP 1
C     POB2 = FRACTION OF FISSION NEUTRONS ( NEVER SPEND TIME
C           IN THE REFLECTOR ) WHICH ARE ABSORBED BY THE
C           CORE AS GROUP 2 , AND SO ON.
C
C     PPB1 = FRACTION OF FISSION NEUTRONS ( SPEND TIME IN
C           THE REFLECTOR ) WHICH ARE ABSORBED BY THE
C           CORE AS GROUP 1
C     PPB2 = FRACTION OF FISSION NEUTRONS ( SPEND TIME IN
C           THE REFLECTOR ) WHICH ARE ABSORBED BY THE
C           CORE AS GROUP 2 , AND SO ON.
C
C     EFFECTIVE NEUTRON MULTIPLICATION FACTOR ( SEE
C     TABLE 5-10 )
C
C     EKEF = EFFECTIVE NEUTRON MULTIPLICATION FACTOR
C     **********************************************************
C
      WRITE(02,100)R,DF
      WRITE(02,102)CA11,CA12,CA13,CA14
      WRITE(02,104)CA22,CA23,CA24
      WRITE(02,106)CA33,CA34,CA44
      WRITE(02,108)RA11,RA12,RA13,RA14
      WRITE(02,110)RA22,RA23,RA24
      WRITE(02,112)RA33,RA34,RA44
      WRITE(02,114)PO1,PO2,PO3,PO4
      WRITE(02,116)SI1,SI2,SI3,SI4
      WRITE(02,118)PR11,PR12,PR13,PR14
      WRITE(02,120)PR22,PR23,PR24
      WRITE(02,122)PR33,PR34,PR44
      WRITE(02,124)POB1,POB2,POB3,POB4
      WRITE(02,126)PPB1,PPB2,PPB3,PPB4
      WRITE(02,128)EKEF
  100 FORMAT(3X,'R   =',E12.5,1X,'DF  =',E12.5)
  102 FORMAT(3X,'CA11=',E12.5,1X,'CA12=',E12.5,1X,'CA13=',E12.5,
     #1X,'CA14=',E12.5)
  104 FORMAT(3X,'CA22=',E12.5,1X,'CA23=',E12.5,1X,'CA24=',E12.5)
  106 FORMAT(3X,'CA33=',E12.5,1X,'CA34=',E12.5,1X,'CA44=',E12.5)
  108 FORMAT(3X,'RA11=',E12.5,1X,'RA12=',E12.5,1X,'RA13=',E12.5,
     #1X,'RA14=',E12.5)
  110 FORMAT(3X,'RA22=',E12.5,1X,'RA23=',E12.5,1X,'RA24=',E12.5)
  112 FORMAT(3X,'RA33=',E12.5,1X,'RA34=',E12.5,1X,'RA44=',E12.5)
  114 FORMAT(3X,'PO1 =',E12.5,1X,'PO2 =',E12.5 ,1X,'PO3 =',E12.5,
     #1X,'PO4 =',E12.5)
  116 FORMAT(3X,'SI1 =',E12.5,1X,'SI2 =',E12.5,1X,'SI3 =',E12.5,
     #1X,'SI4 =',E12.5)
  118 FORMAT(3X,'PR11=',E12.5,1X,'PR12=',E12.5,1X,'PR13=',E12.5,
     #1X,'PR14=',E12.5)
  120 FORMAT(3X,'PR22=',E12.5,1X,'PR23=',E12.5,1X,'PR24=',E12.5)
  122 FORMAT(3X,'PR33=',E12.5,1X,'PR34=',E12.5,1X,'PR44=',E12.5)
  124 FORMAT(3X,'POB1=',E12.5,1X,'POB2=',E12.5,1X,'POB3=',E12.5,
     #1X,'POB4=',E12.5)
  126 FORMAT(3X,'PPB1=',E12.5,1X,'PPB2=',E12.5,1X,'PPB3=',E12.5,
     #1X,'PPB4=',E12.5)
  128 FORMAT(3X,'EFFECTIVE MULTIPLICATION FACTOR =',E12.5)
  999 STOP
      END
```

Source Listing of ALB2

```
C
C     **********************************************************
C     FOUR-GROUP ALBEDOS , NONLEAKAGE PROBABILITIES AND EFFECTIVE
C     NEUTRON MULTIPLICATION FACTOR FOR A SMALL OPTICAL-PATH-
C     LENGTH CORE REACTOR (TWO-REGION GAS CORE REACTOR)
C     **********************************************************
C
      PROGRAM ALB2
      NTIN=01
      NTOUT=02
      OPEN(NTIN, FILE='ALB2.IN', STATUS='OLD')
      OPEN(NTOUT, FILE='ALB2.OUT', STATUS='UNKNOWN')
C
C     **********************************************************
C                         INPUT DATA
```

```
C
C      R     = RADIUS OF THE SPHERICAL CORE
C      DF    = CORE DENSITY FACTOR
C      A     = ANGULAR COEFFICIENT OF ANISOTROPIC SECONDARY
C              SOURCE ON THE REFLECTOR-CORE INTERFACE ( SEE
C              TABLE 6-11 )
C
C      CORE GROUP PARAMETERS ( SEE TABLE 6-2 )
C
C      EN1   = MACROSCOPIC (n,2n) CROSS SECTION FOR GROUP 1
C      EA1   = MACROSCOPIC ABSORPTION CROSS SECTION FOR GROUP 1
C      EA2   = MACROSCOPIC ABSORPTION CROSS SECTION FOR GROUP 2 ,
C              AND SO ON.
C      FV1   = ( AVERAGE NUMBER OF FISSION NEUTRONS RELEASED IN A
C              FISSION REACTION ) * ( MACROSCOPIC FISSION CROSS
C              SECTION ) FOR GROUP 1
C      FV2   = ( AVERAGE NUMBER OF FISSION NEUTRONS RELEASED IN A
C              FISSION REACTION ) * ( MACROSCOPIC FISSION CROSS
C              SECTION ) FOR GROUP 2 , AND SO ON.
C      CHI1  = PROBABILITY THAT A FISSION NEUTRON WILL BE BORN
C              WITH AN ENERGY IN GROUP 1
C      CHI2  = PROBABILITY THAT A FISSION NEUTRON WILL BE BORN
C              WITH AN ENERGY IN GROUP 2
C      ES11  = MACROSCOPIC GROUP-TRANSFER CROSS SECTION FROM
C              GROUP 1 TO GROUP 1
C      ES12  = MACROSCOPIC GROUP-TRANSFER CROSS SECTION FROM
C              GROUP 1 TO GROUP 2 , AND SO ON.
C
C      REFLECTOR GROUP PARAMETERS ( SEE TABLE 6-3 )
C
C      DR1   = DIFFUSION COEFFICIENT FOR GROUP 1
C      DR2   = DIFFUSION COEFFICIENT FOR GROUP 2 , AND SO ON.
C      ERN1  = MACROSCOPIC (n,2n) CROSS SECTION FOR GROUP 1
C      ERA1  = MACROSCOPIC ABSORPTION CROSS SECTION FOR GROUP 1
C      ERA2  = MACROSCOPIC ABSORPTION CROSS SECTION FOR GROUP 2 ,
C              AND SO ON.
C      ERS12= MACROSCOPIC GROUP-TRANSFER CROSS SECTION FROM
C              GROUP 1 TO GROUP 2
C      ERS13= MACROSCOPIC GROUP-TRANSFER CROSS SECTION FROM
C              GROUP 1 TO GROUP 3 , AND SO ON.
C      ************************************************************
C
       READ(01,10)R,DF,A
       READ(01,11)EN1,EA1,EA2,EA3,EA4,FV1,FV2,FV3,FV4,ES11,ES12,
      #ES13,ES14,ES22,ES23,ES24,ES33,ES34,ES44,CHI1,CHI2,DR1,DR2,
      #DR3,DR4,ERN1,ERA1,ERA2,ERA3,ERA4,ERS12,ERS13,ERS14,ERS23,
      #ERS24,ERS34
 10    FORMAT(4E17.7)
 11    FORMAT(4E17.7)
       D1=D1/DF
       D2=D2/DF
       D3=D3/DF
       D4=D4/DF
       ET1=(EA1+EN1+ES11+ES12+ES13+ES14)*DF
       ET2=(EA2+ES22+ES23+ES24)*DF
       ET3=(EA3+ES33+ES34)*DF
       ET4=(EA4+ES44)*DF
       EA1=(EA1+EN1)*DF
       EA2=EA2*DF
       EA3=EA3*DF
       EA4=EA4*DF
       ES11=ES11*DF
       ES12=ES12*DF
       ES13=ES13*DF
       ES14=ES14*DF
       ES22=ES22*DF
       ES23=ES23*DF
       ES24=ES24*DF
       ES33=ES33*DF
       ES34=ES34*DF
       ES44=ES44*DF
       FV1=FV1*DF
       FV2=FV2*DF
       FV3=FV3*DF
       FV4=FV4*DF
       ERR1=ERA1+ERS12+ERS13+ERS14-ERN1
       ERR2=ERA2+ERS23+ERS24
       ERR3=ERA3+ERS34
       ERR4=ERA4
```

```
      RK1=SQRT(ERR1/DR1)
      RK2=SQRT(ERR2/DR2)
      RK3=SQRT(ERR3/DR3)
      RK4=SQRT(ERR4/DR4)
C
C     **********************************************************
C     REFLECTOR ALBEDO CALCULATION
C     **********************************************************
C
      Z1=EXP(-RK1*R)/R
      Z2=EXP(-RK2*R)/R
      Z3=EXP(-RK3*R)/R
      Z4=EXP(-RK4*R)/R
      G34=RK1+1./R
      G35=RK2+1./R
      G36=RK3+1./R
      G37=RK4+1./R
      RA11=(1.-2.*DR1*G34)/(1.+2.*DR1*G34)
      RA22=(1.-2.*DR2*G35)/(1.+2.*DR2*G35)
      RA33=(1.-2.*DR3*G36)/(1.+2.*DR3*G36)
      RA44=(1.-2.*DR4*G37)/(1.+2.*DR4*G37)
      G38=4./(Z1*(1.+2.*DR1*G34))
      G39=G38*ERS12/(DR2*(RK2*RK2-RK1*RK1))
      G40=-G39*Z1*(1.+2.*DR2*G34)/(Z2*(1.+2.*DR2*G35))
      G41=(G38*ERS13+G39*ERS23)/(DR3*(RK3*RK3-RK1*RK1))
      G42=G40*ERS23/(DR3*(RK3*RK3-RK2*RK2))
      G43=-(G41*Z1*(1.+2.*DR3*G34)+G42*Z2*(1.+2.*DR3*G35))/(Z3*
     #(1.+2.*DR3*G36))
      G44=(G38*ERS14+G39*ERS24+G41*ERS34)/(DR4*(RK4*RK4-RK1*RK1))
      G45=(G40*ERS24+G42*ERS34)/(DR4*(RK4*RK4-RK2*RK2))
      G46=G43*ERS34/(DR4*(RK4*RK4-RK3*RK3))
      G47=-(G44*Z1*(1.+2.*DR4*G34)+G45*Z2*(1.+2.*DR4*G35)+G46*Z3*
     #(1.+2.*DR4*G36))/(Z4*(1.+2.*DR4*G37))
      RA12=(G40*Z2+G39*Z1)/2.
      RA13=(G43*Z3+G41*Z1+G42*Z2)/2.
      RA14=(G47*Z4+G44*Z1+G45*Z2+G46*Z3)/2.
      G48=4./(Z2*(1.+2.*DR2*G35))
      G49=G48*ERS23/(DR3*(RK3*RK3-RK2*RK2))
      G50=-G49*Z2*(1.+2.*DR3*G35)/(Z3*(1.+2.*DR3*G36))
      G51=(G48*ERS24+G49*ERS34)/(DR4*(RK4*RK4-RK2*RK2))
      G52=G50*ERS34/(DR4*(RK4*RK4-RK3*RK3))
      G53=-(G51*Z2*(1.+2.*DR4*G35)+G52*Z3*(1.+2.*DR4*G36))/(Z4*
     #(1.+2.*DR4*G37))
      RA23=(G50*Z3+G49*Z2)/2.
      RA24=(G53*Z4+G51*Z2+G52*Z3)/2.
      G54=4./(Z3*(1.+2.*DR3*G36))
      G55=G54*ERS34/(DR4*(RK4*RK4-RK3*RK3))
      G56=-G55*Z3*(1.+2.*DR4*G36)/(Z4*(1.+2.*DR4*G37))
      RA34=(G56*Z4+G55*Z3)/2.
      RO=R/(2.**(1./3))
      S1=CHI1*RA11
      S2=CHI1*RA12+CHI2*RA22
      S3=CHI1*RA13+CHI2*RA23
      S4=CHI1*RA14+CHI2*RA24
C
C     **********************************************************
C     TRANSMITTANCE CALCULATION FOR THE REFLECTOR SECONDARY
C     SOURCES
C     **********************************************************
C
      C1=-2.*ET1*R
      C2=-2.*ET2*R
      C3=-2.*ET3*R
      C4=-2.*ET4*R
      TEF1=(EXP(C1)*(1.+A-A/C1)+A/C1-1.)/(C1*(1.+A/2.))
      TEF2=(EXP(C2)*(1.+A-A/C2)+A/C2-1.)/(C2*(1.+A/2.))
      TEF3=(EXP(C3)*(1.+A-A/C3)+A/C3-1.)/(C3*(1.+A/2.))
      TEF4=(EXP(C4)*(1.+A-A/C4)+A/C4-1.)/(C4*(1.+A/2.))
      X1=(1.-TEF1)/(1.-TEF1*RA11)
      X2=(1.-TEF2)/(1.-TEF2*RA22)
      X3=(1.-TEF3)/(1.-TEF3*RA33)
      X4=(1.-TEF4)/(1.-TEF4*RA44)
      X5=TEF1/(1.-TEF1*RA11)
      X6=TEF2/(1.-TEF2*RA22)
      X7=TEF3/(1.-TEF3*RA33)
C
C     **********************************************************
C     TRANSMITTANCE CALCULATION FOR THE CORE SECONDARY
C     SOURCES
```

```
C     ************************************************************
C
      N=180
      H=3.14159/FLOAT(N)
      NN=N-1
      XX=0.0
      DO 100 K=1,NN
      Y1=FLOAT(K)*H
      Y2=SIN(Y1)
      Y3=RO*COS(Y1)+SQRT(R*R-RO*RO*Y2*Y2)
      Y4=EXP(-ET1*Y3)*Y2
      XX=XX+Y4
 100  CONTINUE
      FFT1=H*XX/2.
      YY=0.0
      DO 101 L=1,NN
      Y5=FLOAT(L)*H
      Y6=SIN(Y5)
      Y7=RO*COS(Y5)+SQRT(R*R-RO*RO*Y6*Y6)
      Y8=EXP(-ET2*Y7)*Y6
      YY=YY+Y8
 101  CONTINUE
      FFT2=H*YY/2.
      ZZ=0.0
      DO 102 I=1,NN
      Y9=FLOAT(I)*H
      Y10=SIN(Y9)
      Y11=RO*COS(Y9)+SQRT(R*R-RO*RO*Y10*Y10)
      Y12=EXP(-ET3*Y11)*Y10
      ZZ=ZZ+Y12
 102  CONTINUE
      FFT3=H*ZZ/2.
      WW=0.0
      DO 103 J=1,NN
      Y13=FLOAT(J)*H
      Y14=SIN(Y13)
      Y15=RO*COS(Y13)+SQRT(R*R-RO*RO*Y14*Y14)
      Y16=EXP(-ET4*Y15)*Y14
      WW=WW+Y16
 103  CONTINUE
      FFT4=H*WW/2.
C
C     ************************************************************
C     CALCULATION OF THE NEUTRON ABSORPTION BY THE CORE
C     ************************************************************
C
      Y17=1.-FFT4+FFT4*RA44*X4
      Y18=Y17*ES44/ET4
      X30=(Y17*EA4/ET4)/(1.-Y18)
      XX4=X4*EA4/ET4+(X4*ES44/ET4)*X30
      X31=(1.-FFT3)+FFT3*RA33*X3
      X32=FFT3*RA34/(1.-TEF3*RA33)
      X33=X31*ES34/ET3
      X34=X31*ES33/ET3
      X35=(X32*XX4+X33*X30)/(1.-X34)
      X36=(X31*EA3/ET3)/(1.-X34)
      X37=X7*RA34
      X38=X3*ES34/ET3
      X39=X3*ES33/ET3
      X40=X37*XX4+X38*X30+X39*X35
      X41=X3*EA3/ET3+X39*X36
      X42=FFT2*RA24/(1.-TEF2*RA22)
      X43=FFT2*RA23/(1.-TEF2*RA22)
      X44=1.-FFT2+FFT2*RA22*X2
      X45=X44*ES24/ET2
      X46=X44*ES23/ET2
      X47=X44*ES22/ET2
      X48=(X42*XX4+X45*X30+X43*X40+X46*X35)/(1.-X47)
      X49=(X43*X41+X46*X36)/(1.-X47)
      X50=(X44*EA2/ET2)/(1.-X47)
      X51=X6*RA24
      X52=X6*RA23
      X53=X2*ES24/ET2
      X54=X2*ES23/ET2
      X55=X2*ES22/ET2
      X56=X51*XX4+X53*X30+X52*X40+X54*X35+X55*X48
      X57=X52*X41+X54*X36+X55*X49
      X58=X2*EA2/ET2+X56*X50
      X59=FFT1*RA14/(1.-TEF1*RA11)
```

```
      X60=FFT1*RA13/(1.-TEF1*RA11)
      X61=FFT1*RA12/(1.-TEF1*RA11)
      X62=1.-FFT1+FFT1*RA11*X1
      X63=X62*ES14/ET1
      X64=X62*ES13/ET1
      X65=X62*ES12/ET1
      X66=X62*ES11/ET1
      X67=(X59*XX4+X63*X30+X60*X40+X64*X35+X61*X56+X65*X48)/(1.-
     #X66)
      X68=(X60*X41+X64*X36+X61*X57+X65*X49)/(1.-X66)
      X69=(X61*X58+X65*X50)/(1.-X66)
      X70=(X62*EA1/ET1)/(1.-X66)
      X71=X5*RA14
      X72=X5*RA13
      X73=X5*RA12
      X74=X1*ES14/ET1
      X75=X1*ES13/ET1
      X76=X1*ES12/ET1
      X77=X1*ES11/ET1
      X78=X71*XX4+X74*X30+X72*X40+X75*X35+X73*X56+X76*X48+X77*X67
      X79=X72*X41+X75*X36+X73*X57+X76*X49+X77*X68
      X80=X73*X58+X76*X50+X77*X69
      X81=X1*EA1/ET1+X77*X70
      X82=S1*X81
      X83=S1*X80+S2*X58
      X84=S1*X79+S2*X57+S3*X41
      X85=S1*X78+S2*X56+S3*X40+S4*XX4
      X86=X82+X83+X84+X85
      POX=0.0
      XKEF=X82*(FV1+2.*EN1)/EA1+X83*FV2/EA2+X84*FV3/EA3+X85*
     #FV4/EA4
      X87=CHI1*X70
      X88=CHI1*X69+CHI2*X50
      X89=CHI1*X68+CHI2*X49
      X90=CHI1*X67+CHI2*X48
      X91=X87+X88+X89+X90
      Q500=(1.-FFT4)*EA4/ET4
      Q501=(1.-FFT4)*ES44/ET4
      Q502=Q500/(1.-Q501)
      Q503=(1.-FFT3)*EA3/ET3
      Q504=(1.-FFT3)*ES33/ET3
      Q505=(1.-FFT3)*ES34/ET3
      Q506=Q503/(1.-Q504)
      Q507=Q505/(1.-Q504)
      Q508=Q507*Q502
      Q509=(1.-FFT2)*EA2/ET2
      Q510=(1.-FFT2)*ES22/ET2
      Q511=(1.-FFT2)*ES23/ET2
      Q512=(1.-FFT2)*ES24/ET2
      Q513=Q509/(1.-Q510)
      Q514=Q511/(1.-Q510)
      Q515=Q512/(1.-Q510)
      Q516=Q515*Q502+Q514*Q508
      Q517=Q514*Q506
      Q518=(1.-FFT1)*EA1/ET1
      Q519=(1.-FFT1)*ES11/ET1
      Q520=(1.-FFT1)*ES12/ET1
      Q521=(1.-FFT1)*ES13/ET1
      Q522=(1.-FFT1)*ES14/ET1
      Q523=Q518/(1.-Q519)
      Q524=Q520/(1.-Q519)
      Q525=Q521/(1.-Q519)
      Q526=Q522/(1.-Q519)
      Q527=Q524*Q513
      Q528=Q524*Q517+Q525*Q506
      Q529=Q524*Q516+Q525*Q508+Q526*Q502
      Q530=CHI1*Q523
      Q531=CHI1*Q527+CHI2*Q513
      Q532=CHI1*Q528+CHI2*Q517
      Q533=CHI1*Q529+CHI2*Q516
      Q534=Q530+Q531+Q532+Q533
      Q535=X91-Q534
      Q540=X87-Q530
      Q541=X88-Q531
      Q542=X89-Q532
      Q543=X90-Q533
C
C     ***********************************************************
C     EFFECTIVE NEUTRON MULTIPLICATION FACTOR CALCULATION
```

```
C     *********************************************************
C
      EKEF=X87*(FV1+2.*EN1)/EA1+X88*FV2/EA2+X89*FV3/EA3+X90*
     #FV4/EA4
C
C     *********************************************************
C                           OUTPUT RESULTS
C
C     REFLECTOR ALBEDOS ( SEE TABLE 6-4 )
C
C     RA11 = PROBABILITY THAT A NEUTRON FROM GROUP 1 BE REFLECTED
C            WITH GROUP 1 ENERGY
C     RA12 = PROBABILITY THAT A NEUTRON FROM GROUP 1 BE REFLECTED
C            WITH GROUP 2 ENERGY , AND SO ON.
C
C     CORE TRANSMITTANCES ( SEE TABLE 6-5 )
C
C     TEF1 = GAMMA1
C     TEF2 = GAMMA2 , AND SO ON.
C     FFT1 = DELTA1
C     FFT2 = DELTA2 , AND SO ON.
C
C     NEUTRON ABSORPTION ( SEE TABLE 6-6 )
C
C     Q530 = FRACTION OF FISSION NEUTRONS ( NEVER SPEND TIME IN
C            THE REFLECTOR ) WHICH ARE ABSORBED BY THE CORE
C            AS GROUP 1
C     Q531 = FRACTION OF FISSION NEUTRONS ( NEVER SPEND TIME IN
C            THE REFLECTOR ) WHICH ARE ABSORBED BY THE CORE
C            AS GROUP 2
C     Q532 = FRACTION OF FISSION NEUTRONS ( NEVER SPEND TIME IN
C            THE REFLECTOR ) WHICH ARE ABSORBED BY THE CORE
C            AS GROUP 3
C     Q533 = FRACTION OF FISSION NEUTRONS ( NEVER SPEND TIME IN
C            THE REFLECTOR ) WHICH ARE ABSORBED BY THE CORE
C            AS GROUP 4
C     Q540 = FRACTION OF FISSION NEUTRONS ( SPEND TIME IN THE
C            REFLECTOR ) WHICH ARE ABSORBED BY THE CORE
C            AS GROUP 1
C     Q541 = FRACTION OF FISSION NEUTRONS ( SPEND TIME IN THE
C            REFLECTOR ) WHICH ARE ABSORBED BY THE CORE
C            AS GROUP 2
C     Q542 = FRACTION OF FISSION NEUTRONS ( SPEND TIME IN THE
C            REFLECTOR ) WHICH ARE ABSORBED BY THE CORE
C            AS GROUP 3
C     Q543 = FRACTION OF FISSION NEUTRONS ( SPEND TIME IN THE
C            REFLECTOR ) WHICH ARE ABSORBED BY THE CORE
C            AS GROUP 4
C
C     EFFECTIVE NEUTRON MULTIPLICATION FACTOR ( SEE TABLE 6-7 )
C
C     EKEF = EFFECTIVE NEUTRON MULTIPLICATION FACTOR
C     *********************************************************
C
      WRITE(02,200)R,DF,A
      WRITE(02,202)RA11,RA12,RA13,RA14
      WRITE(02,204)RA22,RA23,RA24
      WRITE(02,206)RA33,RA34,RA44
      WRITE(02,208)TEF1,TEF2,TEF3,TEF4
      WRITE(02,210)FFT1,FFT2,FFT3,FFT4
      WRITE(02,212)Q530,Q531,Q532,Q533
      WRITE(02,214)Q540,Q541,Q542,Q543
      WRITE(02,216)EKEF
 200  FORMAT(3X,'R   =',E12.5,1X,'DF  =',E12.5,1X,'A   =',E12.5)
 202  FORMAT(3X,'RA11=',E12.5,1X,'RA12=',E12.5,1X,'RA13=',E12.5,
     #1X,'RA14=',E12.5)
 204  FORMAT(3X,'RA22=',E12.5,1X,'RA23=',E12.5,1X,'RA24=',E12.5)
 206  FORMAT(3X,'RA33=',E12.5,1X,'RA34=',E12.5,1X,'RA44=',E12.5)
 208  FORMAT(3X,'TEF1=',E12.5,1X,'TEF2=',E12.5,1X,'TEF3=',E12.5,
     #1X,'TEF4=',E12.5)
 210  FORMAT(3X,'FFT1=',E12.5,1X,'FFT2=',E12.5,1X,'FFT3=',E12.5,
     #1X,'FFT4=',E12.5)
 212  FORMAT(3X,'Q530=',E12.5,1X,'Q531=',E12.5,1X,'Q532=',E12.5,
     #1X,'Q533=',E12.5)
 214  FORMAT(3X,'Q540=',E12.5,1X,'Q541=',E12.5,1X,'Q542=',E12.5,
     #1X,'Q543=',E12.5)
 216  FORMAT(3X,'EFFECTIVE MULTIPLICATION FACTOR',9X,'=',E12.5)
      STOP
      END
```

Source Listing of ALB3

```
C
C     ************************************************************
C     TRANSPORT - THEORY REFLECTOR ALBEDOS
C     ( P3 - PLANE SYMMETRY / FOUR-GROUP ENERGY )
C     ************************************************************
C
      PROGRAM ALB3
      NTIN=01
      NTOUT=02
      OPEN(NTIN, FILE='ALB3.IN', STATUS='OLD')
      OPEN(NTOUT, FILE='ALB3.OUT', STATUS='UNKNOWN')
C
C     ************************************************************
C                        INPUT DATA
C
C     B     = ANGULAR COEFFICIENT FOR AN ANISOTROPIC SOURCE
C             ON THE REFLECTOR-CORE INTERFACE FOR REFLECTOR
C             ALBEDO CALCULATION
C
C     REFLECTOR GROUP PARAMETERS ( SEE TABLE 6-3 )
C
C     DR1   = DIFFUSION COEFFICIENT FOR GROUP 1
C     DR2   = DIFFUSION COEFFICIENT FOR GROUP 2 , AND SO ON.
C     ENR1  = MACROSCOPIC (n,2n) CROSS SECTION FOR GROUP 1
C     ERA1  = MACROSCOPIC ABSORPTION CROSS SECTION FOR GROUP 1
C     ERT2  = MACROSCOPIC TOTAL CROSS SECTION FOR GROUP 2
C     ERT3  = MACROSCOPIC TOTAL CROSS SECTION FOR GROUP 3 ,
C             AND SO ON.
C     ERS11= MACROSCOPIC GROUP-TRANSFER CROSS SECTION FROM
C             GROUP 1 TO GROUP 1
C     ERS13= MACROSCOPIC GROUP-TRANSFER CROSS SECTION FROM
C             GROUP 1 TO GROUP 3 , AND SO ON.
C     ************************************************************
C
      READ(01,10)B
      READ(01,11)DR1,DR2,DR3,DR4,ENR1,ERA1,ERT2,ERT3,ERT4,ERS11,
     #ERS12,ERS13,ERS14,ERS22,ERS23,ERS24,ERS33,ERS34,ERS44
 10   FORMAT(E17.7)
 11   FORMAT(4E17.7)
      ERR1=ERA1+ERS12+ERS13+ERS14-ERN1
      ERR2=ERT2-ERS22
      ERR3=ERT3-ERS33
      ERR4=ERT4-ERS44
      ERT1=ERA1+ERN1+ERS11+ERS12+ERS13+ERS14
C
C     ************************************************************
C     REFLECTOR ALBEDO CALCULATION
C     ************************************************************
C
      HH=1.+B/2.
      HI=(4.+3.*B)/12.
      X30=35.*ERT1*ERT1+55.*ERT1*ERR1
      X31=SQRT(1225.*ERT1**4+3025.*ERT1*ERT1*ERR1*ERR1+70.*ERR1*
     #ERT1**3)
      X32=(X30+X31)/18.
      XN1=SQRT(X32)
      X33=(X30-X31)/18.
      XN2=SQRT(X33)
      X34=35.*ERT2*ERT2+55.*ERT2*ERR2
      X35=SQRT(1225.*ERT2**4+3025.*ERT2*ERT2*ERR2*ERR2+70.*ERR2*
     #ERT2**3)
      X36=(X34+X35)/18.
      XM1=SQRT(X36)
      X37=(X34-X35)/18.
      XM2=SQRT(X37)
      X38=35.*ERT3*ERT3+55.*ERT3*ERR3
      X39=SQRT(1225.*ERT3**4+3025.*ERT3*ERT3*ERR3*ERR3+70.*ERR3*
     #ERT3**3)
      X40=(X38+X39)/18.
      XP1=SQRT(X40)
      X41=(X38-X39)/18.
      XP2=SQRT(X41)
      X42=35.*ERT4*ERT4+55.*ERT4*ERR4
      X43=SQRT(1225.*ERT4**4+3025.*ERT4*ERT4*ERR4*ERR4+70.*ERR4*
     #ERT4**3)
```

```
      X44=(X42+X43)/18.
      XQ1=SQRT(X44)
      X45=(X42-X43)/18.
      XQ2=SQRT(X45)
      X50=ERR1/XN1
      X51=ERR1/XN2
      X52=(3.*ERT1*X50-XN1)/(2.*XN1)
      X53=(3.*ERT1*X51-XN2)/(2.*XN2)
      X54=(3.*XN1*X52)/(7.*ERT1)
      X55=(3.*XN2*X53)/(7.*ERT1)
      X56=10.+24.*X50+25.*X52+16.*X54
      X57=10.+24.*X51+25.*X53+16.*X55
      X58=8.*X50+5.*X52+4.
      X59=8.*X51+5.*X53+4.
      X60=(80.*HI*X59-16.*HH*X57)/(X56*X59-X57*X58)
      X61=(16.*HH*X56-80.*HI*X58)/(X56*X59-X57*X58)
      X62=((0.5-X50+5.*X52/8.)*X60+(0.5-X51+5.*X53/8.)*X61)/2.
      RA11=X62/HH
      SQ1=35.*ERT2*ERT2+55.*ERT2*ERR2
      SQ2=105.*ERR2*ERT2**3
      SQ3=35.*ERT3*ERT3+55.*ERT3*ERR3
      SQ4=105.*ERR3*ERT3**3
      SQ5=35.*ERT4*ERT4+55.*ERT4*ERR4
      SQ6=105.*ERR4*ERT4**3
      X100=9.*XN1**4-SQ1*XN1**2+SQ2
      X101=9.*XN2**4-SQ1*XN2**2+SQ2
      X102=9.*XN1**4-SQ3*XN1**2+SQ4
      X103=9.*XN2**4-SQ3*XN2**2+SQ4
      X104=9.*XM1**4-SQ3*XM1**2+SQ4
      X105=9.*XM2**4-SQ3*XM2**2+SQ4
      X106=9.*XN1**4-SQ5*XN1**2+SQ6
      X107=9.*XN2**4-SQ5*XN2**2+SQ6
      X108=9.*XM1**4-SQ5*XM1**2+SQ6
      X109=9.*XM2**4-SQ5*XM2**2+SQ6
      X110=9.*XP1**4-SQ5*XP1**2+SQ6
      X111=9.*XP2**4-SQ5*XP2**2+SQ6
      X63=(35.*XN1*ERT2**2-9.*XN1**3)*ERS12*X60
      X64=X63/X100
      X65=(35.*XN2*ERT2**2-9.*XN2**3)*ERS12*X61
      X66=X65/X101
      X67=14.*ERT2*ERS12*X60*XN1*XN1/X100
      X68=14.*ERT2*ERS12*X61*XN2*XN2/X101
      X69=(ERS12*X60+XN1*X64)/ERR2
      X70=(ERS12*X61+XN2*X66)/ERR2
      X71=3.*XN1*X67/(7.*ERT2)
      X72=3.*XN2*X68/(7.*ERT2)
      X73=(3.*ERT2-XM1*XM1/ERR2)/(2.*XM1)
      X74=(3.*ERT2-XM2*XM2/ERR2)/(2.*XM2)
      X75=XM1/ERR2
      X76=XM2/ERR2
      X77=3.*XM1*X73/(7.*ERT2)
      X78=3.*XM2*X74/(7.*ERT2)
      X79=4.*X75+8.+5.*X73
      X80=4.*X76+8.+5.*X74
      X81=-8.*(X64+X66)-5.*(X67+X68)-4.*(X69+X70)
      X82=10.*X75+24.+25.*X73+16.*X77
      X83=10.*X76+24.+25.*X74+16.*X78
      X84=-24.*(X64+X66)-25.*(X67+X68)-10.*(X69+X70)-16.*(X71+
     #X72)
      X85=X79*X83-X80*X82
      X86=(X81*X83-X80*X84)/X85
      X87=(X79*X84-X81*X82)/X85
      X88=((X75*X86+X76*X87+X69+X70)/2.-(X86+X87+X64+X66)+5.*(
     #X73*X86+X74*X87+X67+X68)/8.)/2.
      RA12=X88/HH
      X89=XM1*X86/ERR2
      X90=XM2*X87/ERR2
      X91=(-9.*XN1**3+35.*XN1*ERT3**2)*(ERS13*X60+ERS23*X69)
      X92=X91/X102
      X93=(-9.*XN2**3+35.*XN2*ERT3**2)*(ERS13*X61+ERS23*X70)
      X94=X93/X103
      X95=(-9.*XM1**3+35.*XM1*ERT3**2)*ERS23*X89/X104
      X96=(-9.*XM2**3+35.*XM2*ERT3**2)*ERS23*X90/X105
      Y10=14.*ERT3*(ERS13*X60+ERS23*X69)*XN1**2/X102
      Y11=14.*ERT3*(ERS13*X61+ERS23*X70)*XN2**2/X103
      Y12=14.*ERT3*ERS23*X89*XM1**2/X104
      Y13=14.*ERT3*ERS23*X90*XM2**2/X105
      Y14=(ERS13*X60+ERS23*X69+XN1*X92)/ERR3
      Y15=(ERS13*X61+ERS23*X70+XN2*X94)/ERR3
```

```
      Y16=(ERS23*X89+XM1*X95)/ERR3
      Y17=(ERS23*X90+XM2*X96)/ERR3
      Y18=3.*Y10*XN1/(7.*ERT3)
      Y19=3.*Y11*XN2/(7.*ERT3)
      Y20=3.*Y12*XM1/(7.*ERT3)
      Y21=3.*Y13*XM2/(7.*ERT3)
      Y22=XP1/ERR3
      Y23=XP2/ERR3
      Y24=(3.*ERT3-XP1*XP1/ERR3)/(2.*XP1)
      Y25=(3.*ERT3-XP2*XP2/ERR3)/(2.*XP2)
      Y26=3.*XP1*Y24/(7.*ERT3)
      Y27=3.*XP2*Y25/(7.*ERT3)
      Y28=4.*Y22+8.+5.*Y24
      Y29=4.*Y23+8.+5.*Y25
      Y30=-4.*(Y14+Y15+Y16+Y17)-8.*(X92+X94+X95+X96)-5.*(Y10+
     #Y11+Y12+Y13)
      Y31=10.*Y22+24.+25.*Y24+16.*Y26
      Y32=10.*Y23+24.+25.*Y25+16.*Y27
      Y33=-10.*(Y14+Y15+Y16+Y17)-24.*(X92+X94+X95+X96)-25.*(Y10+
     #Y11+Y12+Y13)-16.*(Y18+Y19+Y20+Y21)
      Y34=Y28*Y32-Y29*Y31
      Y35=(Y30*Y32-Y29*Y33)/Y34
      Y36=(Y28*Y33-Y30*Y31)/Y34
      Y37=((Y22*Y35+Y23*Y36+Y14+Y15+Y16+Y17)/2.-(Y35+Y36+X92+X94+
     #X95+X96)+5.*(Y24*Y35+Y25*Y36+Y10+Y11+Y12+Y13)/8.)/2.
      RA13=Y37/HH
      Y38=XP1*Y35/ERR3
      Y39=XP2*Y36/ERR3
      Y40=(-9.*XN1**3+35.*XN1*ERT4**2)*(ERS34*Y14+ERS24*X69+
     #ERS14*X60)
      Y41=Y40/X106
      Y42=(-9.*XN2**3+35.*XN2*ERT4**2)*(ERS34*Y15+ERS24*X70+
     #ERS14*X61)
      Y43=Y42/X107
      Y44=(-9.*XM1**3+35.*XM1*ERT4**2)*(ERS34*Y16+ERS24*X89)
      Y45=Y44/X108
      Y46=(-9.*XM2**3+35.*XM2*ERT4**2)*(ERS34*Y17+ERS24*X90)
      Y47=Y46/X109
      Y48=(-9.*XP1**3+35.*XP1*ERT4**2)*ERS34*Y38/X110
      Y49=(-9.*XP2**3+35.*XP2*ERT4**2)*ERS34*Y39/X111
      Y50=14.*ERT4*(ERS34*Y14+ERS24*X69+ERS14*X60)*XN1**2
      Y51=Y50/X106
      Y52=14.*ERT4*(ERS34*Y15+ERS24*X70+ERS14*X61)*XN2**2
      Y53=Y52/X107
      Y54=14.*ERT4*(ERS34*Y16+ERS24*X89)*XM1**2
      Y55=Y54/X108
      Y56=14.*ERT4*(ERS34*Y17+ERS24*X90)*XM2**2
      Y57=Y56/X109
      Y58=14.*ERT4*ERS34*Y38*XP1**2/X110
      Y59=14.*ERT4*ERS34*Y39*XP2**2/X111
      Y60=(ERS34*Y14+ERS24*X69+ERS14*X60+Y41*XN1)/ERR4
      Y61=(ERS34*Y15+ERS24*X70+ERS14*X61+Y43*XN2)/ERR4
      Y62=(ERS34*Y16+ERS24*X89+Y45*XM1)/ERR4
      Y63=(ERS34*Y17+ERS24*X90+Y47*XM2)/ERR4
      Y64=(ERS34*Y38+Y48*XP1)/ERR4
      Y65=(ERS34*Y39+Y49*XP2)/ERR4
      YY65=3.*Y51*XN1/(7.*ERT4)
      Y66=3.*Y53*XN2/(7.*ERT4)
      Y67=3.*Y55*XM1/(7.*ERT4)
      Y68=3.*Y57*XM2/(7.*ERT4)
      Y69=3.*Y58*XP1/(7.*ERT4)
      Y70=3.*Y59*XP2/(7.*ERT4)
      Y71=XQ1/ERR4
      Y72=XQ2/ERR4
      Y73=(3.*ERT4-XQ1*XQ1/ERR4)/(2.*XQ1)
      Y74=(3.*ERT4-XQ2*XQ2/ERR4)/(2.*XQ2)
      Y75=3.*XQ1*Y73/(7.*ERT4)
      Y76=3.*XQ2*Y74/(7.*ERT4)
      Y78=4.*Y71+8.+5.*Y73
      Y79=4.*Y72+8.+5.*Y74
      Y80=-8.*(Y41+Y43+Y45+Y47+Y48+Y49)-5.*(Y51+Y53+Y55+Y57+
     #Y58+Y59)-4.*(Y60+Y61+Y62+Y63+Y64+Y65)
      Y81=10.*Y71+24.+25.*Y73+16.*Y75
      Y82=10.*Y72+24.+25.*Y74+16.*Y76
      Y83=-24.*(Y41+Y43+Y45+Y47+Y48+Y49)-25.*(Y51+Y53+Y55+Y57+
     #Y58+Y59)-10.*(Y60+Y61+Y62+Y63+Y64+Y65)-16.*(YY65+Y66+Y67+
     #Y68+Y69+Y70)
      Y84=Y78*Y82-Y79*Y81
      Y85=(Y80*Y82-Y79*Y83)/Y84
```

```
 Y86=(Y78*Y83-Y80*Y81)/Y84
 Y87=((Y71*Y85+Y72*Y86+Y60+Y61+Y62+Y63+Y64+Y65)/2.-(Y85+
#Y86+Y41+Y43+Y45+Y47+Y48+Y49)+5.*(Y73*Y85+Y74*Y86+Y51+Y53+
#Y55+Y57+Y58+Y59)/8.)/2.
 RA14=Y87/HH
 W10=ERR2/XM1
 W11=ERR2/XM2
 W12=(3.*ERT2*W10-XM1)/(2.*XM1)
 W13=(3.*ERT2*W11-XM2)/(2.*XM2)
 W14=3.*XM1*W12/(7.*ERT2)
 W15=3.*XM2*W13/(7.*ERT2)
 W16=10.+24.*W10+25.*W12+16.*W14
 W17=10.+24.*W11+25.*W13+16.*W15
 W18=8.*W10+5.*W12+4.
 W19=8.*W11+5.*W13+4.
 W20=W16*W19-W17*W18
 W21=16.*(5.*W19*HI-W17*HH)/W20
 W22=16.*(W16*HH-5.*W18*HI)/W20
 W23=((0.5-W10+5.*W12/8.)*W21+(0.5-W11+5.*W13/8.)*W22)/2.
 RA22=W23/HH
 W24=(35.*XM1*ERT3**2-9.*XM1**3)*ERS23*W21/X104
 W25=(35.*XM2*ERT3**2-9.*XM2**3)*ERS23*W22/X105
 W26=14.*ERT3*ERS23*W21*XM1**2/X104
 W27=14.*ERT3*ERS23*W22*XM2**2/X105
 W28=(ERS23*W21+XM1*W24)/ERR3
 W29=(ERS23*W22+XM2*W25)/ERR3
 W30=3.*XM1*W26/(7.*ERT3)
 W31=3.*XM2*W27/(7.*ERT3)
 W32=(3.*ERT3-XP1*XP1/ERR3)/(2.*XP1)
 W33=(3.*ERT3-XP2*XP2/ERR3)/(2.*XP2)
 W34=XP1/ERR3
 W35=XP2/ERR3
 W36=3.*XP1*W32/(7.*ERT3)
 W37=3.*XP2*W33/(7.*ERT3)
 W38=4.*W34+8.+5.*W32
 W39=4.*W35+8.+5.*W33
 W40=-8.*(W24+W25)-5.*(W26+W27)-4.*(W28+W29)
 W41=10.*W34+24.+25.*W32+16.*W36
 W42=10.*W35+24.+25.*W33+16.*W37
 W43=-24.*(W24+W25)-25.*(W26+W27)-10.*(W28+W29)-16.*(
#W30+W31)
 W44=W38*W42-W39*W41
 W45=(W40*W42-W39*W43)/W44
 W46=(W38*W43-W40*W41)/W44
 W47=((W34*W45+W35*W46+W28+W29)/2.-(W45+W46+W24+W25)+5.*(
#W32*W45+W33*W46+W26+W27)/8.)/2.
 RA23=W47/HH
 W48=W34*W45
 W49=W35*W46
 W50=(-9.*XM1**3+35.*XM1*ERT4**2)*(ERS24*W21+ERS34*W28)/X108
 W51=(-9.*XM2**3+35.*XM2*ERT4**2)*(ERS24*W22+ERS34*W29)/X109
 W52=(-9.*XP1**3+35.*XP1*ERT4**2)*ERS34*W48/X110
 W53=(-9.*XP2**3+35.*XP2*ERT4**2)*ERS34*W49/X111
 W54=14.*ERT4*XM1**2*(ERS24*W21+ERS34*W28)/X108
 W55=14.*ERT4*XM2**2*(ERS24*W22+ERS34*W29)/X109
 W56=14.*ERT4*ERS34*W48*XP1**2/X110
 W57=14.*ERT4*ERS34*W49*XP2**2/X111
 W58=(ERS24*W21+ERS34*W28+XM1*W50)/ERR4
 W59=(ERS24*W22+ERS34*W29+XM2*W51)/ERR4
 W60=(ERS34*W48+XP1*W52)/ERR4
 W61=(ERS34*W49+XP2*W53)/ERR4
 W62=3.*XM1*W54/(7.*ERT4)
 W63=3.*XM2*W55/(7.*ERT4)
 W64=3.*XP1*W56/(7.*ERT4)
 W65=3.*XP2*W57/(7.*ERT4)
 W66=(3.*ERT4-XQ1*XQ1/ERR4)/(2.*XQ1)
 W67=(3.*ERT4-XQ2*XQ2/ERR4)/(2.*XQ2)
 W68=XQ1/ERR4
 W69=XQ2/ERR4
 W70=3.*XQ1*W66/(7.*ERT4)
 W71=3.*XQ2*W67/(7.*ERT4)
 W72=4.*W68+8.+5.*W66
 W73=4.*W69+8.+5.*W67
 W74=-8.*(W50+W51+W52+W53)-5.*(W54+W55+W56+W57)-4.*(
#W58+W59+W60+W61)
 W75=10.*W68+24.+25.*W66+16.*W70
 W76=10.*W69+24.+25.*W67+16.*W71
 W77=-24.*(W50+W51+W52+W53)-25.*(W54+W55+W56+W57)-10.*(
#W58+W59+W60+W61)-16.*(W62+W63+W64+W65)
```

```
      W78=W72*W76-W73*W75
      W79=(W74*W76-W73*W77)/W78
      W80=(W72*W77-W74*W75)/W78
      W81=((W68*W79+W69*W80+W58+W59+W60+W61)/2.-(W79+W80+W50+
     #W51+W52+W53)+5.*(W66*W79+W67*W80+W54+W55+W56+W57)/8.)/2.
      RA24=W81/HH
      Z10=ERR3/XP1
      Z11=ERR3/XP2
      Z12=(3.*ERT3*Z10-XP1)/(2.*XP1)
      Z13=(3.*ERT3*Z11-XP2)/(2.*XP2)
      Z14=3.*XP1*Z12/(7.*ERT3)
      Z15=3.*XP2*Z13/(7.*ERT3)
      Z16=10.+24.*Z10+25.*Z12+16.*Z14
      Z17=10.+24.*Z11+25.*Z13+16.*Z15
      Z18=8.*Z10+5.*Z12+4.
      Z19=8.*Z11+5.*Z13+4.
      Z20=Z16*Z19-Z17*Z18
      Z21=16.*(5.*HI*Z19-HH*Z17)/Z20
      Z22=16.*(HH*Z16-5.*HI*Z18)/Z20
      Z23=((0.5-Z10+5.*Z12/8.)*Z21+(0.5-Z11+5.*Z13/8.)*Z22)/2.
      RA33=Z23/HH
      Z24=(35.*XP1*ERT4**2-9.*XP1**3)*ERS34*Z21/X110
      Z25=(35.*XP2*ERT4**2-9.*XP2**3)*ERS34*Z22/X111
      Z26=14.*ERT4*ERS34*Z21*XP1**2/X110
      Z27=14.*ERT4*ERS34*Z22*XP2**2/X111
      Z28=(ERS34*Z21+XP1*Z24)/ERR4
      Z29=(ERS34*Z22+XP2*Z25)/ERR4
      Z30=3.*XP1*Z26/(7.*ERT4)
      Z31=3.*XP2*Z27/(7.*ERT4)
      Z32=XQ1/ERR4
      Z33=XQ2/ERR4
      Z34=(3.*ERT4-XQ1*XQ1/ERR4)/(2.*XQ1)
      Z35=(3.*ERT4-XQ2*XQ2/ERR4)/(2.*XQ2)
      Z36=3.*XQ1*Z34/(7.*ERT4)
      Z37=3.*XQ2*Z35/(7.*ERT4)
      Z38=4.*Z32+8.+5.*Z34
      Z39=4.*Z33+8.+5.*Z35
      Z40=-8.*(Z24+Z25)-5.*(Z26+Z27)-4.*(Z28+Z29)
      Z41=10.*Z32+24.+25.*Z34+16.*Z36
      Z42=10.*Z33+24.+25.*Z35+16.*Z37
      Z43=-24.*(Z24+Z25)-25.*(Z26+Z27)-10.*(Z28+Z29)-16.*(
     #Z30+Z31)
      Z44=Z38*Z42-Z39*Z41
      Z45=(Z40*Z42-Z39*Z43)/Z44
      Z46=(Z38*Z43-Z40*Z41)/Z44
      Z47=((Z32*Z45+Z33*Z46+Z28+Z29)/2.-(Z45+Z46+Z24+Z25)+5.*(
     #Z34*Z45+Z35*Z46+Z26+Z27)/8.)/2.
      RA34=Z47/HH
      Z48=ERR4/XQ1
      Z49=ERR4/XQ2
      Z50=(3.*ERT4*Z48-XQ1)/(2.*XQ1)
      Z51=(3.*ERT4*Z49-XQ2)/(2.*XQ2)
      Z52=3.*XQ1*Z50/(7.*ERT4)
      Z53=3.*XQ2*Z51/(7.*ERT4)
      Z54=10.+24.*Z48+25.*Z50+16.*Z52
      Z55=10.+24.*Z49+25.*Z51+16.*Z53
      Z56=8.*Z48+5.*Z50+4.
      Z57=8.*Z49+5.*Z51+4.
      Z58=Z54*Z57-Z55*Z56
      Z59=16.*(5.*HI*Z57-HH*Z55)/Z58
      Z60=16.*(HH*Z54-5.*HI*Z56)/Z58
      Z61=((0.5-Z48+5.*Z50/8.)*Z59+(0.5-Z49+5.*Z51/8.)*Z60)/2.
      RA44=Z61/HH
C
C     ***********************************************************
C                           OUTPUT RESULTS
C
C     REFLECTOR ALBEDOS (SEE TABLE            )
C
C     RA11 = PROBABILITY THAT A NEUTRON FROM GROUP 1 BE
C            REFLECTED WITH GROUP 1 ENERGY
C     RA12 = PROBABILITY THAT A NEUTRON FROM GROUP 1 BE
C            REFLECTED WITH GROUP 2 ENERGY , AND SO ON.
C     ***********************************************************
C
      WRITE(02,100)B
      WRITE(02,102)RA11,RA12,RA13,RA14
      WRITE(02,104)RA22,RA23,RA24
      WRITE(02,106)RA33,RA34,RA44
```

```
100  FORMAT(3X,'B   =',E12.5)
102  FORMAT(3X,'RA11=',E12.5,1X,'RA12=',E12.5,1X,'RA13=',E12.5,
    #1X,'RA14=',E12.5)
104  FORMAT(3X,'RA22=',E12.5,1X,'RA23=',E12.5,1X,'RA24=',E12.5)
106  FORMAT(3X,'RA33=',E12.5,1X,'RA34=',E12.5,1X,'RA44=',E12.5)
     STOP
     END
```

Source Listing of ALB4

```
C
C     ***********************************************************
C     FOUR-GROUP ALBEDOS, NONLEAKAGE PROBABILITIES AND EFFECTIVE
C     NEUTRON MULTIPLICATION FACTOR FOR A SMALL OPTICAL-PATH-
C     LENGTH CORE REACTOR ( THREE-REGION GAS CORE REACTOR )
C     ***********************************************************
C
      PROGRAM ALB4
      NTIN=01
      NTOUT=02
      OPEN(NTIN, FILE='ALB4.IN', STATUS='OLD')
      OPEN(NTOUT, FILE='ALB4.OUT', STATUS='UNKNOWN')
C
C     ***********************************************************
C                           INPUT DATA
C
C     RP   = OUTER HYDROGEN RADIUS
C     RC   = FUEL RADIUS
C     PDF  = HYDROGEN DENSITY FACTOR
C
C     OSCILLATIONS OF THE FUEL-HYDROGEN BOUNDARY
C     RDEL = DIFFERENCE BETWEEN THE WAVE MAXIMUM AMPLITUDE
C            AND THE UNPERTURBED FUEL RADIUS ( RC )
C
C     NONUNIFORM ATOMIC DENSITY DISTRIBUTION IN THE FUEL
C     REGION ( SEE TABLE 7-9 )
C     AF   = ANGULAR COEFFICIENT OF A LINEAR ATOMIC DENSITY
C            DISTRIBUTION
C
C     FUEL GROUP PARAMETERS ( SEE TABLE 6-2 )
C
C     EN1  = MACROSCOPIC (n,2n) CROSS SECTION FOR GROUP 1
C     EA1  = MACROSCOPIC ABSORPTION CROSS SECTION FOR GROUP 1
C     EA2  = MACROSCOPIC ABSORPTION CROSS SECTION FOR GROUP 2 ,
C            AND SO ON.
C     FV1  = ( AVERAGE NUMBER OF FISSION NEUTRNS RELEASED IN A
C            FISSION REACTION ) * ( MACROSCOPIC FISSION CROSS
C            SECTION ) FOR GROUP 1
C     FV2  = ( AVERAGE NUMBER OF FISSION NEUTRONS RELEASED IN A
C            FISSION REACTION ) * ( MACROSCOPIC FISSION CROSS
C            SECTION ) FOR GROUP 2 ,AND SO ON.
C     ET1  = MACROSCOPIC TOTAL CROSS SECTION FOR GROUP 1
C     ET2  = MACROSCOPIC TOTAL CROSS SECTION FOR GROUP 2 ,
C            AND SO ON.
C     ES11 = MACROSCOPIC GROUP-TRANSFER CROSS SECTION FROM
C            GROUP 1 TO GROUP 1
C     ES12 = MACROSCOPIC GROUP-TRANSFER CROSS SECTION FROM
C            GROUP 1 TO GROUP 2 , AND SO ON.
C     CHI1 = PROBABILITY THAT A FISSION NEUTRON WILL BE BORN
C            WITH AN ENERGY IN GROUP 1
C     CHI2 = PROBABILITY THAT A FISSION NEUTRON WILL BE BORN
C            WITH AN ENERGY IN GROUP 2
C
C     HYDROGEN GROUP PARAMETERS ( SEE TABLE 7-2 )
C
C     EPA1 = MACROSCOPIC ABSORPTION CROSS SECTION FOR GROUP 1
C     EPA2 = MACROSCOPIC ABSORPTION CROSS SECTION FOR GROUP 2 ,
C            AND SO ON.
C     EPS11= MACROSCOPIC GROUP-TRANSFER CROSS SECTION FROM
C            GROUP 1 TO GROUP 1
C     EPS12= MACROSCOPIC GROUP-TRANSFER CROSS SECTION FROM
C            GROUP 1 TO GROUP 2 , AND SO ON.
C
C     REFLECTOR GROUP PARAMETERS ( SEE TABLE 6-3 )
C
```

```
C     DR1  = DIFFUSION COEFFICIENT FOR GROUP 1
C     DR2  = DIFFUSION COEFFICIENT FOR GROUP 2 , AND SO ON.
C     ERN1 = MACROSCOPIC (n,2n) CROSS SECTION FOR GROUP 1
C     ERA1 = MACROSCOPIC ABSORPTION CROSS SECTION FOR GROUP 1
C     ERA2 = MACROSCOPIC ABSORPTION CROSS SECTION FOR GROUP 2 ,
C            AND SO ON.
C     ERS12= MACROSCOPIC GROUP-TRANSFER CROSS SECTION FROM
C            GROUP 1 TO GROUP 2
C     ERS13= MACROSCOPIC GROUP-TRANSFER CROSS SECTION FROM
C            GROUP 1 TO GROUP 3 , AND SO ON.
C     ************************************************************
C
      READ(01,10)RP,RC
      READ(01,11)PDF,RDEL,AF
      READ(01,12)EN1,EA1,EA2,EA3,EA4,FV1,FV2,FV3,FV4,ET1,ET2,ET3,
     #ET4,ES11,ES12,ES13,ES14,ES22,ES23,ES24,ES33,ES34,ES44,CHI1,
     #CHI2,EPA1,EPA2,EPA3,EPA4,EPS11,EPS12,EPS13,EPS14,EPS22,
     #EPS23,EPS24,EPS33,EPS34,EPS44,DR1,DR2,DR3,DR4,ERN1,ERA1,
     #ERA2,ERA3,ERA4,ERS12,ERS13,ERS14,ERS23,ERS24,ERS34
   10 FORMAT(2E17.7)
   11 FORMAT(3E17.7)
   12 FORMAT(4E17.7)
      ERR1=ERA1+ERS12+ERS13+ERS14-ERN1
      ERR2=ERA2+ERS23+ERS24
      ERR3=ERA3+ERS34
      ERR4=ERA4
C
C     ************************************************************
C     REFLECTOR ALBEDO CALCULATION
C     ************************************************************
C
      RK1=SQRT(ERR1/DR1)
      RK2=SQRT(ERR2/DR2)
      RK3=SQRT(ERR3/DR3)
      RK4=SQRT(ERR4/DR4)
      Z1=EXP(-RK1*RP)/RP
      Z2=EXP(-RK2*RP)/RP
      Z3=EXP(-RK3*RP)/RP
      Z4=EXP(-RK4*RP)/RP
      G34=RK1+1./RP
      G35=RK2+1./RP
      G36=RK3+1./RP
      G37=RK4+1./RP
      RA11=(1.-2.*DR1*G34)/(1.+2.*DR1*G34)
      RA22=(1.-2.*DR2*G35)/(1.+2.*DR2*G35)
      RA33=(1.-2.*DR3*G36)/(1.+2.*DR3*G36)
      RA44=(1.-2.*DR4*G37)/(1.+2.*DR4*G37)
      G38=4./(Z1*(1.+2.*DR1*G34))
      G39=G38*ERS12/(DR2*(RK2*RK2-RK1*RK1))
      G40=-G39*Z1*(1.+2.*DR2*G34)/(Z2*(1.+2.*DR2*G35))
      G41=(G38*ERS13+G39*ERS23)/(DR3*(RK3*RK3-RK1*RK1))
      G42=G40*ERS23/(DR3*(RK3*RK3-RK2*RK2))
      G43=-(G41*Z1*(1.+2.*DR3*G34)+G42*Z2*(1.+2.*DR3*G35))/(Z3*(
     #1.+2.*DR3*G36))
      G44=(G38*ERS14+G39*ERS24+G41*ERS34)/(DR4*(RK4*RK4-RK1*RK1))
      G45=(G40*ERS24+G42*ERS34)/(DR4*(RK4*RK4-RK2*RK2))
      G46=G43*ERS34/(DR4*(RK4*RK4-RK3*RK3))
      G47=-(G44*Z1*(1.+2.*DR4*G34)+G45*Z2*(1.+2.*DR4*G35)+G46*Z3*
     #(1.+2.*DR4*G36))/(Z4*(1.+2.*DR4*G37))
      RA12=(G40*Z2+G39*Z1)/2.
      RA13=(G43*Z3+G41*Z1+G42*Z2)/2.
      RA14=(G47*Z4+G44*Z1+G45*Z2+G46*Z3)/2.
      G48=4./(Z2*(1.+2.*DR2*G35))
      G49=G48*ERS23/(DR3*(RK3*RK3-RK2*RK2))
      G50=-G49*Z2*(1.+2.*DR3*G35)/(Z3*(1.+2.*DR3*G36))
      G51=(G48*ERS24+G49*ERS34)/(DR4*(RK4*RK4-RK2*RK2))
      G52=G50*ERS34/(DR4*(RK4*RK4-RK3*RK3))
      G53=-(G51*Z2*(1.+2.*DR4*G35)+G52*Z3*(1.+2*DR4*G36))/(Z4*(
     #1.+2.*DR4*G37))
      RA23=(G50*Z3+G49*Z2)/2.
      RA24=(G53*Z4+G51*Z2+G52*Z3)/2.
      G54=4./(Z3*(1.+2.*DR3*G36))
      G55=G54*ERS34/(DR4*(RK4*RK4-RK3*RK3))
      G56=-G55*Z3*(1.+2.*DR4*G36)/(Z4*(1.+2.*DR4*G37))
      RA34=(G56*Z4+G55*Z3)/2.
C
C     ************************************************************
C     CALCULATION OF FUEL AND HYDROGEN TRANSMITTANCES
C     ************************************************************
```

```
C
      EPT1=EPA1+EPS11+EPS12+EPS13+EPS14
      EPT2=EPA2+EPS22+EPS23+EPS24
      EPT3=EPA3+EPS33+EPS34
      EPT4=EPA4+EPS44
      EPA1=PDF*EPA1
      EPA2=PDF*EPA2
      EPA3=PDF*EPA3
      EPA4=PDF*EPA4
      EPS11=PDF*EPS11
      EPS12=PDF*EPS12
      EPS13=PDF*EPS13
      EPS14=PDF*EPS14
      EPS22=PDF*EPS22
      EPS23=PDF*EPS23
      EPS24=PDF*EPS24
      EPS33=PDF*EPS33
      EPS34=PDF*EPS34
      EPS44=PDF*EPS44
      EPT1=PDF*EPT1
      EPT2=PDF*EPT2
      EPT3=PDF*EPT3
      EPT4=PDF*EPT4
      EA1=EA1+EN1
      FR=1.0
      WO=ASIN(FR*RC/RP)
      A=0.0
      N=90
      H=(WO-A)/FLOAT(N)
      NN=N-1
      AFF=(1.+AF*RC/2.)/(1.+0.75*AF*RC)
      FF=AFF/(FR*FR*FR)
      FP=(RP**3-RC**3)/(RP**3-(FR*RC)**3)
      SAP=SQRT(RP*RP-FR*FR*RC*RC)/RP
      SAC=1.-SAP
      SX1=FR*RC/RP
      XX=0.0
      DO 40 K=1,NN
      X1=A+FLOAT(K)*H
      Q1=2.*FF*ET1*SQRT(FR*FR*RC*RC-RP*RP*SIN(X1)*SIN(X1))
      X=EXP(-Q1)*SIN(X1)
      XX=XX+X
 40   CONTINUE
      EF1=H*(SX1/2.+XX)/SAC
      SY1=FR*RC/RP
      YY=0.0
      DO 42 J=1,NN
      Y1=A+FLOAT(J)*H
      Q2=2.*FF*ET2*SQRT(FR*FR*RC*RC-RP*RP*SIN(Y1)*SIN(Y1))
      Y=EXP(-Q2)*SIN(Y1)
      YY=YY+Y
 42   CONTINUE
      EF2=H*(SY1/2.+YY)/SAC
      SW1=FR*RC/RP
      WW=0.0
      DO 44 I=1,NN
      W1=A+FLOAT(I)*H
      Q3=2.*FF*ET3*SQRT(FR*FR*RC*RC-RP*RP*SIN(W1)*SIN(W1))
      W=EXP(-Q3)*SIN(W1)
      WW=WW+W
 44   CONTINUE
      EF3=H*(SW1/2.+WW)/SAC
      SZ1=FR*RC/RP
      ZZ=0.0
      DO 46 L=1,NN
      Z5=A+FLOAT(L)*H
      Q4=2.*FF*ET4*SQRT(FR*FR*RC*RC-RP*RP*SIN(Z5)*SIN(Z5))
      Z=EXP(-Q4)*SIN(Z5)
      ZZ=ZZ+Z
 46   CONTINUE
      EF4=H*(SZ1/2.+ZZ)/SAC
      C4=RP*SAP
      TX1=EXP(-EPT1*FP*C4)*FR*RC/RP
      XX1=0.0
      DO 48 KK=1,NN
      X11=A+FLOAT(KK)*H
      H2=FP*EPT1*(RP*COS(X11)-SQRT(FR*FR*RC*RC-RP*RP*SIN(X11)
     #**2))
      X2=EXP(-H2)*SIN(X11)
```

```
      XX1=XX1+X2
 48   CONTINUE
      EPP1=H*(TX1/2.+XX1)/SAC
      TX2=EXP(-EPT2*FP*C4)*FR*RC/RP
      XX2=0.0
      DO 50 JJ=1,NN
      X12=A+FLOAT(JJ)*H
      H3=FP*EPT2*(RP*COS(X12)-SQRT(FR*FR*RC*RC-RP*RP*SIN(X12)
     #**2))
      X3=EXP(-H3)*SIN(X12)
      XX2=XX2+X3
 50   CONTINUE
      EPP2=H*(TX2/2.+XX2)/SAC
      TX3=EXP(-EPT3*FP*C4)*FR*RC/RP
      XX3=0.0
      DO 52 LL=1,NN
      X13=A+FLOAT(LL)*H
      H4=FP*EPT3*(RP*COS(X13)-SQRT(FR*FR*RC*RC-RP*RP*SIN(X13)
     #**2))
      X4=EXP(-H4)*SIN(X13)
      XX3=XX3+X4
 52   CONTINUE
      EPP3=H*(TX3/2.+XX3)/SAC
      TX4=EXP(-EPT4*FP*C4)*FR*RC/RP
      XX4=0.0
      DO 54 II=1,NN
      X14=A+FLOAT(II)*H
      H5=FP*EPT4*(RP*COS(X14)-SQRT(FR*FR*RC*RC-RP*RP*SIN(X14)
     #**2))
      X5=EXP(-H5)*SIN(X14)
      XX4=XX4+X5
 54   CONTINUE
      EPP4=H*(TX4/2.+XX4)/SAC
      W11=FP*EPT1*2.*RP*SAP
      OPP1=(1.-EXP(-W11))/W11
      W12=FP*EPT2*2.*RP*SAP
      OPP2=(1.-EXP(-W12))/W12
      W13=FP*EPT3*2.*RP*SAP
      OPP3=(1.-EXP(-W13))/W13
      W14=FP*EPT4*2.*RP*SAP
      OPP4=(1.-EXP(-W14))/W14
      H11=1.-(SAC*EPP4*EPP4*EF4+SAP*OPP4)*RA44
      H12=1.-(SAC*EPP3*EPP3*EF3+SAP*OPP3)*RA33
      H13=1.-(SAC*EPP2*EPP2*EF2+SAP*OPP2)*RA22
      H14=1.-(SAC*EPP1*EPP1*EF1+SAP*OPP1)*RA11
      RCC=FR*RC
      RO=((RP**3+RCC**3)/2.)**(1./3.)
      IF(AF)601,600,601
 600  ROC=RCC/(2.**(1./3))
      GO TO 602
 601  PI=3.14159
      A1=4./(3.*AF)
      A4=-2.*RCC*RCC*RCC*(1./3.+AF*RCC/4.)/AF
      CA2=-4.*A4
      CA3=-A1*A1*A4
      P1=CA2/3.
      P2=-CA3/2.
      P3=P1**3+P2*P2
      P4=P2+(ABS(P3))**0.5
      P5=P2-(ABS(P3))**0.5
      P6=(ABS(P4))**(1./3.)
      P7=(ABS(P5))**(1./3.)
      IF(P3)610,652,652
 610  TETA=ACOS(P2/(-P1**3)**0.5)
      RQ1=2.*(-P1)**0.5*COS(TETA/3.)
      RQ2=2.*(-P1)**0.5*COS(TETA/3.+2.*PI/3.)
      RQ3=2.*(-P1)**0.5*COS(TETA/3.+4.*PI/3.)
      DSR1=A1*A1+4.*RQ1
      ESR1=RQ1*RQ1-4.*A4
      IF(DSR1)625,620,620
 620  CONTINUE
      IF(ESR1)625,622,622
 622  XR1=RQ1
      GO TO 664
 625  DSR2=A1*A1+4.*RQ2
      ESR2=RQ2*RQ2-4.*A4
      IF(DSR2)635,630,630
 630  CONTINUE
      IF(ESR2)635,632,632
```

```
632  XR1=RQ2
     GO TO 664
635  DSR3=A1*A1+4.*RQ3
     ESR3=RQ3*RQ3-4.*A4
     IF(DSR3)999,640,640
640  CONTINUE
     IF(ESR3)999,642,642
642  XR1=RQ3
     GO TO 664
652  CONTINUE
     IF(P4)654,656,656
654  P6=-P6
     GO TO 657
656  P6=P6
657  CONTINUE
     IF(P5)658,660,660
658  P7=-P7
     GO TO 661
660  P7=P7
661  CONTINUE
     XR1=P6+P7
664  DIS=A1*A1+4.*XR1
     EIS=XR1*XR1-4.*A4
     IF(DIS)999,666,666
666  CONTINUE
     IF(EIS)999,667,667
667  CONTINUE
     P8=(A1-SQRT(DIS))/2.
     P9=(XR1+SQRT(EIS))/2.
     P10=(A1+SQRT(DIS))/2.
     P11=(XR1-SQRT(EIS))/2.
     DIS1=P8*P8-4.*P9
     P12=(ABS(DIS1))**0.5
     IF(DIS1)681,670,670
670  ZZ1=(-P8+P12)/2.
     ZZ2=(-P8-P12)/2.
     IF(ZZ1)673,673,671
671  CONTINUE
     IF(ZZ1-RCC)672,673,673
672  ROC=ZZ1
     GO TO 602
673  CONTINUE
     IF(ZZ2)681,681,674
674  CONTINUE
     IF(ZZ2-RCC)676,681,681
676  ROC=ZZ2
     GO TO 602
681  DIS2=P10*P10-4.*P11
     P13=(ABS(DIS2))**0.5
     IF(DIS2)999,682,682
682  ZZ3=(-P10+P13)/2.
     ZZ4=(-P10-P13)/2.
     IF(ZZ3)685,685,683
683  CONTINUE
     IF(ZZ3-RCC)684,685,685
684  ROC=ZZ3
     GO TO 602
685  CONTINUE
     IF(ZZ4)999,999,686
686  CONTINUE
     IF(ZZ4-RCC)687,999,999
687  ROC=ZZ4
     GO TO 602
602  CONTINUE
     BB=2.*ASIN(1.)
     NK=150
     NK1=NK-1
     R1=RO
     R2=RO
     R3=RO
     R4=RO
     U1=ASIN(FR*RC/R1)
     HH1=(BB-U1)/FLOAT(NK)
     SU1=FR*RC/R1
     CU1=SQRT(1.-SU1*SU1)
     SG1=R1*CU1+SQRT(RP*RP-R1*R1*SU1*SU1)
     PX1=EXP(-FP*EPT1*SG1)*SU1
     PP1=0.0
     DO 200 K9=1,NK1
```

```
      X30=U1+FLOAT(K9)*HH1
      X31=SIN(X30)
      X32=R1*COS(X30)+SQRT(RP*RP-R1*R1*X31*X31)
      X33=EXP(-FP*EPT1*X32)*X31
      PP1=PP1+X33
200   CONTINUE
      XPP1=HH1*(PX1/2.+PP1)/(1.+CU1)
      U2=ASIN(FR*RC/R2)
      HH2=(BB-U2)/FLOAT(NK)
      SU2=FR*RC/R2
      CU2=SQRT(1.-SU2*SU2)
      SG2=R2*CU2+SQRT(RP*RP-R2*R2*SU2*SU2)
      PX2=EXP(-FP*EPT2*SG2)*SU2
      PP2=0.0
      DO 201 K10=1,NK1
      X35=U2+FLOAT(K10)*HH2
      X36=SIN(X35)
      X37=R2*COS(X35)+SQRT(RP*RP-R2*R2*X36*X36)
      X38=EXP(-FP*EPT2*X37)*X36
      PP2=PP2+X38
201   CONTINUE
      XPP2=HH2*(PX2/2.+PP2)/(1.+CU2)
      U3=ASIN(FR*RC/R3)
      HH3=(BB-U3)/FLOAT(NK)
      SU3=FR*RC/R3
      CU3=SQRT(1.-SU3*SU3)
      SG3=R3*CU3+SQRT(RP*RP-R3*R3*SU3*SU3)
      PX3=EXP(-FP*EPT3*SG3)*SU3
      PP3=0.0
      DO 202 K11=1,NK1
      X40=U3+FLOAT(K11)*HH3
      X41=SIN(X40)
      X42=R3*COS(X40)+SQRT(RP*RP-R3*R3*X41*X41)
      X43=EXP(-FP*EPT3*X42)*X41
      PP3=PP3+X43
202   CONTINUE
      XPP3=HH3*(PX3/2.+PP3)/(1.+CU3)
      U4=ASIN(FR*RC/R4)
      HH4=(BB-U4)/FLOAT(NK)
      SU4=FR*RC/R4
      CU4=SQRT(1.-SU4*SU4)
      SG4=R4*CU4+SQRT(RP*RP-R4*R4*SU4*SU4)
      PX4=EXP(-FP*EPT4*SG4)*SU4
      PP4=0.0
      DO 203 K12=1,NK1
      X45=U4+FLOAT(K12)*HH4
      X46=SIN(X45)
      X47=R4*COS(X45)+SQRT(RP*RP-R4*R4*X46*X46)
      X48=EXP(-FP*EPT4*X47)*X46
      PP4=PP4+X48
203   CONTINUE
      XPP4=HH4*(PX4/2.+PP4)/(1.+CU4)
      HL1=U1/FLOAT(NK)
      SQ1=R1*CU1
      PX5=EXP(-FP*EPT1*SQ1)*SU1
      PP5=0.0
      DO 204 K13=1,NK1
      X50=A+FLOAT(K13)*HL1
      X51=SIN(X50)
      X52=R1*COS(X50)-SQRT(RCC*RCC-R1*R1*X51*X51)
      X53=EXP(-FP*EPT1*X52)*X51
      PP5=PP5+X53
204   CONTINUE
      YP11=HL1*(PX5/2.+PP5)/(1.-CU1)
      HL2=U2/FLOAT(NK)
      SQ2=R2*CU2
      PX6=EXP(-FP*EPT2*SQ2)*SU2
      PP6=0.0
      DO 205 K14=1,NK1
      X55=A+FLOAT(K14)*HL2
      X56=SIN(X55)
      X57=R2*COS(X55)-SQRT(RCC*RCC-R2*R2*X56*X56)
      X58=EXP(-FP*EPT2*X57)*X56
      PP6=PP6+X58
205   CONTINUE
      YP12=HL2*(PX6/2.+PP6)/(1.-CU2)
      HL3=U3/FLOAT(NK)
      SQ3=R3*CU3
      PX7=EXP(-FP*EPT3*SQ3)*SU3
```

```
      PP7=0.0
      DO 206 K15=1,NK1
      X60=A+FLOAT(K15)*HL3
      X61=SIN(X60)
      X62=R3*COS(X60)-SQRT(RCC*RCC-R3*R3*X61*X61)
      X63=EXP(-FP*EPT3*X62)*X61
      PP7=PP7+X63
206   CONTINUE
      YP13=HL3*(PX7/2.+PP7)/(1.-CU3)
      HL4=U4/FLOAT(NK)
      SQ4=R4*CU4
      PX8=EXP(-FP*EPT4*SQ4)*SU4
      PP8=0.0
      DO 207 K16=1,NK1
      X65=A+FLOAT(K16)*HL4
      X66=SIN(X65)
      X67=R4*COS(X65)-SQRT(RCC*RCC-R4*R4*X66*X66)
      X68=EXP(-FP*EPT4*X67)*X66
      PP8=PP8+X68
207   CONTINUE
      YP14=HL4*(PX8/2.+PP8)/(1.-CU4)
      SMNO=SQRT(RP*RP-RCC*RCC)
      PX9=EXP(-FP*EPT1*SMNO)*SU1
      PP9=0.0
      DO 208 K17=1,NK1
      X70=A+FLOAT(K17)*HL1
      X71=SIN(X70)
      X72=SQRT(RP*RP-R1*R1*X71*X71)-SQRT(RCC*RCC-R1*R1*X71*X71)
      X73=EXP(-FP*EPT1*X72)*X71
      PP9=PP9+X73
208   CONTINUE
      YP21=HL1*(PX9/2.+PP9)/(1.-CU1)
      PX10=EXP(-FP*EPT2*SMNO)*SU2
      PP10=0.0
      DO 209 K18=1,NK1
      X75=A+FLOAT(K18)*HL2
      X76=SIN(X75)
      X77=SQRT(RP*RP-R2*R2*X76*X76)-SQRT(RCC*RCC-R2*R2*X76*X76)
      X78=EXP(-FP*EPT2*X77)*X76
      PP10=PP10+X78
209   CONTINUE
      YP22=HL2*(PX10/2.+PP10)/(1.-CU2)
      PX11=EXP(-FP*EPT3*SMNO)*SU3
      PP11=0.0
      DO 210 K19=1,NK1
      X80=A+FLOAT(K19)*HL3
      X81=SIN(X80)
      X82=SQRT(RP*RP-R3*R3*X81*X81)-SQRT(RCC*RCC-R3*R3*X81*X81)
      X83=EXP(-FP*EPT3*X82)*X81
      PP11=PP11+X83
210   CONTINUE
      YP23=HL3*(PX11/2.+PP11)/(1.-CU3)
      PX12=EXP(-FP*EPT4*SMNO)*SU4
      PP12=0.0
      DO 211 K20=1,NK1
      X85=A+FLOAT(K20)*HL4
      X86=SIN(X85)
      X87=SQRT(RP*RP-R4*R4*X86*X86)-SQRT(RCC*RCC-R4*R4*X86*X86)
      X88=EXP(-FP*EPT4*X87)*X86
      PP12=PP12+X88
211   CONTINUE
      YP24=HL4*(PX12/2.+PP12)/(1.-CU4)
      PX13=SU1
      PP13=0.0
      DO 212 K21=1,NK1
      X90=A+FLOAT(K21)*HL1
      X91=SIN(X90)
      X92=-FF*ET1*2.*SQRT(RCC*RCC-R1*R1*X91*X91)
      X93=EXP(+X92)*X91
      PP13=PP13+X93
212   CONTINUE
      YCC1=HL1*(PX13/2.+PP13)/(1.-CU1)
      PX14=SU2
      PP14=0.0
      DO 213 K22=1,NK1
      X95=A+FLOAT(K22)*HL2
      X96=SIN(X95)
      X97=-FF*ET2*2.*SQRT(RCC*RCC-R2*R2*X96*X96)
      X98=EXP(+X97)*X96
```

```
      PP14=PP14+X98
  213 CONTINUE
      YCC2=HL2*(PX14/2.+PP14)/(1.-CU2)
      PX15=SU3
      PP15=0.0
      DO 214 K23=1,NK1
      X100=A+FLOAT(K23)*HL3
      X101=SIN(X100)
      X102=-FF*ET3*2.*SQRT(RCC*RCC-R3*R3*X101*X101)
      X103=EXP(+X102)*X101
      PP15=PP15+X103
  214 CONTINUE
      YCC3=HL3*(PX15/2.+PP15)/(1.-CU3)
      PX16=SU4
      PP16=0.0
      DO 215 K24=1,NK1
      X105=A+FLOAT(K24)*HL4
      X106=SIN(X105)
      X107=-FF*ET4*2.*SQRT(RCC*RCC-R4*R4*X106*X106)
      X108=EXP(+X107)*X106
      PP16=PP16+X108
  215 CONTINUE
      YCC4=HL4*(PX16/2.+PP16)/(1.-CU4)
      I1=180
      HI=2.*ASIN(1.)/FLOAT(I1)
      I2=I1-1
      XI1=0.0
      DO 220 K1=1,I2
      Y10=FLOAT(K1)*HI
      Y11=SIN(Y10)
      Y12=-FF*ET1*(ROC*COS(Y10)+SQRT(RCC*RCC-ROC*ROC*Y11*Y11))
      Y13=EXP(Y12)*Y11
      XI1=XI1+Y13
  220 CONTINUE
      TC1=HI*XI1/2.
      XI2=0.0
      DO 222 K2=1,I2
      Y15=FLOAT(K2)*HI
      Y16=SIN(Y15)
      Y17=-FF*ET2*(ROC*COS(Y15)+SQRT(RCC*RCC-ROC*ROC*Y16*Y16))
      Y18=EXP(Y17)*Y16
      XI2=XI2+Y18
  222 CONTINUE
      TC2=HI*XI2/2.
      XI3=0.0
      DO 224 K3=1,I2
      Y20=FLOAT(K3)*HI
      Y21=SIN(Y20)
      Y22=-FF*ET3*(ROC*COS(Y20)+SQRT(RCC*RCC-ROC*ROC*Y21*Y21))
      Y23=EXP(Y22)*Y21
      XI3=XI3+Y23
  224 CONTINUE
      TC3=HI*XI3/2.
      XI4=0.0
      DO 226 K4=1,I2
      Y25=FLOAT(K4)*HI
      Y26=SIN(Y25)
      Y27=-FF*ET4*(ROC*COS(Y25)+SQRT(RCC*RCC-ROC*ROC*Y26*Y26))
      Y28=EXP(Y27)*Y26
      XI4=XI4+Y28
  226 CONTINUE
      TC4=HI*XI4/2.
      XI5=0.0
      DO 228 K5=1,I2
      Y30=FLOAT(K5)*HI
      Y31=SIN(Y30)
      Y32=SQRT(RP*RP-ROC*ROC*Y31*Y31)-SQRT(RCC**2-ROC**2*Y31**2)
      Y33=EXP(-FP*EPT1*Y32)*Y31
      XI5=XI5+Y33
  228 CONTINUE
      TP1=HI*XI5/2.
      XI6=0.0
      DO 230 K6=1,I2
      Y35=FLOAT(K6)*HI
      Y36=SIN(Y35)
      Y37=SQRT(RP*RP-ROC*ROC*Y36*Y36)-SQRT(RCC**2-ROC**2*Y36**2)
      Y38=EXP(-FP*EPT2*Y37)*Y36
      XI6=XI6+Y38
  230 CONTINUE
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      TP2=HI*XI6/2.
      XI7=0.0
      DO 232 K7=1,I2
      Y40=FLOAT(K7)*HI
      Y41=SIN(Y40)
      Y42=SQRT(RP*RP-ROC*ROC*Y41*Y41)-SQRT(RCC**2-ROC**2*Y41**2)
      Y43=EXP(-FP*EPT3*Y42)*Y41
      XI7=XI7+Y43
232   CONTINUE
      TP3=HI*XI7/2.
      XI8=0.0
      DO 234 K8=1,I2
      Y45=FLOAT(K8)*HI
      Y46=SIN(Y45)
      Y47=SQRT(RP*RP-ROC*ROC*Y46*Y46)-SQRT(RCC**2-ROC**2*Y46**2)
      Y48=EXP(-FP*EPT4*Y47)*Y46
      XI8=XI8+Y48
234   CONTINUE
      TP4=HI*XI8/2.
      GC1=(1.-CU1)/2.
      GP1=1.-GC1
      GC2=(1.-CU2)/2.
      GP2=1.-GC2
      GC3=(1.-CU3)/2.
      GP3=1.-GC3
      GC4=(1.-CU4)/2.
      GP4=1.-GC4
      IF(RDEL) 500,500,236
236   RCL=RCC+RDEL
      RCS=RCC-RDEL
      RH=RO
      PI=3.14159
      A7=(RCC+RCS)/(2.*RH)
      A8=ASIN(A7)
      A9=COS(A8)
      A10=1.-A9
      GF1=A10/2.
      A11=RCC*COS(PI/8.)
      B11=RCC*SIN(PI/8.)
      A12=RCS*SIN(PI/4.)
      B12=A12
      A13=RCC*COS(3.*PI/8.)
      B13=RCC*SIN(3.*PI/8.)
      A14=(RH+A11)**2+B11*B11
      A15=SQRT(A14)
      A16=(RH+A12)**2+B12*B12
      A17=SQRT(A16)
      A18=(RH+A13)**2+B13*B13
      A19=SQRT(A18)
      A20=SQRT(RH*RH+RCL*RCL)
      A21=ASIN(B11/A15)
      A22=ASIN(B12/A17)
      A23=ASIN(B13/A19)
      A24=ASIN(RCL/A20)
      A25=B11/A15
      A26=B12/A17
      A27=B13/A19
      A28=RCL/A20
      A30=COS(A21)
      A31=COS(A22)
      A32=COS(A23)
      A33=COS(A24)
      A35=RH*A30+SQRT(RP*RP-RH*RH*A25*A25)
      A36=RH*A31+SQRT(RP*RP-RH*RH*A26*A26)
      A37=RH*A32+SQRT(RP*RP-RH*RH*A27*A27)
      A38=RH*A33+SQRT(RP*RP-RH*RH*A28*A28)
      A39=RH*A9+SQRT(RP*RP-RH*RH*A7*A7)
      A40=RH*A30-SQRT(RCL*RCL-RH*RH*A25*A25)
      A41=RH*A31-SQRT(RCL*RCL-RH*RH*A26*A26)
      A42=RH*A32-SQRT(RCL*RCL-RH*RH*A27*A27)
      A43=(RCL+RCC)/2.
      A44=RH*A33-SQRT(A43*A43-RH*RH*A28*A28)
      AA44=SQRT(B11*B11+(RH-A11)*(RH-A11))
      A45=EXP(-EPT1*A40)*A25
      A46=EXP(-EPT1*A41)*A26
      A47=EXP(-EPT1*A42)*A27
      A48=EXP(-EPT1*A44)*A28
      A49=EXP(-EPT1*AA44)*A7
      A50=EXP(-EPT2*A40)*A25
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      A51=EXP(-EPT2*A41)*A26
      A52=EXP(-EPT2*A42)*A27
      A53=EXP(-EPT2*A44)*A28
      A54=EXP(-EPT2*AA44)*A7
      A55=EXP(-EPT3*A40)*A25
      A56=EXP(-EPT3*A41)*A26
      A57=EXP(-EPT3*A42)*A27
      A58=EXP(-EPT3*A44)*A28
      A59=EXP(-EPT3*AA44)*A7
      A60=EXP(-EPT4*A40)*A25
      A61=EXP(-EPT4*A41)*A26
      A62=EXP(-EPT4*A42)*A27
      A63=EXP(-EPT4*A44)*A28
      A64=EXP(-EPT4*AA44)*A7
      A65=(A45*A21+(A45+A46)*(A22-A21)+(A46+A47)*(A23-A22)+(
     #A47+A48)*(A24-A23)+(A48+A49)*(A8-A24))/2.
      T1H1=A65/A10
      A66=(A50*A21+(A50+A51)*(A22-A21)+(A51+A52)*(A23-A22)+(
     #A52+A53)*(A24-A23)+(A53+A54)*(A8-A24))/2.
      T1H2=A66/A10
      A67=(A55*A21+(A55+A56)*(A22-A21)+(A56+A57)*(A23-A22)+(
     #A57+A58)*(A24-A23)+(A58+A59)*(A8-A24))/2.
      T1H3=A67/A10
      A68=(A60*A21+(A60+A61)*(A22-A21)+(A61+A62)*(A23-A22)+(
     #A62+A63)*(A24-A23)+(A63+A64)*(A8-A24))/2.
      T1H4=A68/A10
      A70=A15-A40
      A71=A17-A41
      A72=A19-A42
      A73=A20-A44
      A75=EXP(-ET1*A70)*A25
      A76=EXP(-ET1*A71)*A26
      A77=EXP(-ET1*A72)*A27
      A78=EXP(-ET1*A73)*A28
      A80=EXP(-ET2*A70)*A25
      A81=EXP(-ET2*A71)*A26
      A82=EXP(-ET2*A72)*A27
      A83=EXP(-ET2*A73)*A28
      A85=EXP(-ET3*A70)*A25
      A86=EXP(-ET3*A71)*A26
      A87=EXP(-ET3*A72)*A27
      A88=EXP(-ET3*A73)*A28
      A90=EXP(-ET4*A70)*A25
      A91=EXP(-ET4*A71)*A26
      A92=EXP(-ET4*A72)*A27
      A93=EXP(-ET4*A73)*A28
      A95=(A75*A21+(A75+A76)*(A22-A21)+(A76+A77)*(A23-A22)+(
     #A77+A78)*(A24-A23)+(A78+A7)*(A8-A24))/2.
      T1F1=A95/A10
      A96=(A80*A21+(A80+A81)*(A22-A21)+(A81+A82)*(A23-A22)+(
     #A82+A83)*(A24-A23)+(A83+A7)*(A8-A24))/2.
      T1F2=A96/A10
      A97=(A85*A21+(A85+A86)*(A22-A21)+(A86+A87)*(A23-A22)+(
     #A87+A88)*(A24-A23)+(A88+A7)*(A8-A24))/2.
      T1F3=A97/A10
      A98=(A90*A21+(A90+A91)*(A22-A21)+(A91+A92)*(A23-A22)+(
     #A92+A93)*(A24-A23)+(A93+A7)*(A8-A24))/2.
      T1F4=A98/A10
      A100=A35-A15
      A101=A36-A17
      A102=A37-A19
      A103=A38-A20
      A104=A39-AA44
      A106=EXP(-EPT1*A100)*A25
      A107=EXP(-EPT1*A101)*A26
      A108=EXP(-EPT1*A102)*A27
      A109=EXP(-EPT1*A103)*A28
      A110=EXP(-EPT1*A104)*A7
      A112=EXP(-EPT2*A100)*A25
      A113=EXP(-EPT2*A101)*A26
      A114=EXP(-EPT2*A102)*A27
      A115=EXP(-EPT2*A103)*A28
      A116=EXP(-EPT2*A104)*A7
      A118=EXP(-EPT3*A100)*A25
      A119=EXP(-EPT3*A101)*A26
      A120=EXP(-EPT3*A102)*A27
      A121=EXP(-EPT3*A103)*A28
      A122=EXP(-EPT3*A104)*A7
      A124=EXP(-EPT4*A100)*A25
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      A125=EXP(-EPT4*A101)*A26
      A126=EXP(-EPT4*A102)*A27
      A127=EXP(-EPT4*A103)*A28
      A128=EXP(-EPT4*A104)*A7
      A130=(A106*A21+(A106+A107)*(A22-A21)+(A107+A108)*(A23-A22)
     #+(A108+A109)*(A24-A23)+(A109+A110)*(A8-A24))/2.
      T2H1=A130/A10
      A131=(A112*A21+(A112+A113)*(A22-A21)+(A113+A114)*(A23-A22)
     #+(A114+A115)*(A24-A23)+(A115+A116)*(A8-A24))/2.
      T2H2=A131/A10
      A132=(A118*A21+(A118+A119)*(A22-A21)+(A119+A120)*(A23-A22)
     #+(A120+A121)*(A24-A23)+(A121+A122)*(A8-A24))/2.
      T2H3=A132/A10
      A133=(A124*A21+(A124+A125)*(A22-A21)+(A125+A126)*(A23-A22)
     #+(A126+A127)*(A24-A23)+(A127+A128)*(A8-A24))/2.
      T2H4=A133/A10
      A138=1.+A9
      NNK=150
      NNK1=NNK-1
      HY1=(BB-A8)/FLOAT(NNK)
      A139=EXP(-EPT1*A39)*A7
      A140=EXP(-EPT2*A39)*A7
      A141=EXP(-EPT3*A39)*A7
      A142=EXP(-EPT4*A39)*A7
      A143=0.0
      A144=0.0
      A145=0.0
      A146=0.0
      DO 304 K30=1,NNK1
      A148=A8+FLOAT(K30)*HY1
      A149=SIN(A148)
      A150=COS(A148)
      A151=RH*A150+SQRT(RP*RP-RH*RH*A149*A149)
      A152=EXP(-EPT1*A151)*A149
      A153=EXP(-EPT2*A151)*A149
      A154=EXP(-EPT3*A151)*A149
      A155=EXP(-EPT4*A151)*A149
      A143=A143+A152
      A144=A144+A153
      A145=A145+A154
      A146=A146+A155
304   CONTINUE
      THH1=HY1*(A143+A139/2.)/A138
      THH2=HY1*(A144+A140/2.)/A138
      THH3=HY1*(A145+A141/2.)/A138
      THH4=HY1*(A146+A142/2.)/A138
      A200=-RCL*RCL/RH
      A201=SQRT(-A200*(RH+A200))*RCC/RCL
      A202=SQRT(A201*A201+(RH+A200)*(RH+A200))
      A203=ASIN(A201/A202)
      A204=SIN(A203)
      A205=COS(A203)
      A206=RH*A205+SQRT(RP*RP-RH*RH*A204*A204)
      A207=A206-A202
      A208=1.-A205
      A209=1.+A205
      GF2=A208/2.
      A210=EXP(-EPT1*A202)*A204
      A211=EXP(-EPT2*A202)*A204
      A212=EXP(-EPT3*A202)*A204
      A213=EXP(-EPT4*A202)*A204
      A215=0.0
      A216=0.0
      A217=0.0
      A218=0.0
      HY2=A203/FLOAT(NNK)
      DO 308 K33=1,NNK1
      A220=FLOAT(K33)*HY2
      A221=SIN(A220)
      A222=COS(A220)
      A223=A221/A222
      A224=RH*A223
      A225=A223**2+(RCC/RCL)**2
      A226=(A224-RCC)*(A224+RCC)
      A227=(2.*A223*A224)**2-4.*A225*A226
      A228=SQRT(A227)
      A229=(-2.*A223*A224-A228)/(2.*A225)
      A230=(RH+A229)/A222
      A231=EXP(-EPT1*A230)*A221
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      A232=EXP(-EPT2*A230)*A221
      A233=EXP(-EPT3*A230)*A221
      A234=EXP(-EPT4*A230)*A221
      A215=A215+A231
      A216=A216+A232
      A217=A217+A233
      A218=A218+A234
 308  CONTINUE
      E1H1=HY2*(A215+A210/2.)/A208
      E1H2=HY2*(A216+A211/2.)/A208
      E1H3=HY2*(A217+A212/2.)/A208
      E1H4=HY2*(A218+A213/2.)/A208
      A288=0.0
      A289=0.0
      A290=0.0
      A291=0.0
      DO 316 K37=1,NNK1
      A292=FLOAT(K37)*HY2
      A293=SIN(A292)
      A294=COS(A292)
      A295=A293/A294
      A296=RH*A295
      A297=A295**2+(RCC/RCL)**2
      A298=(A296-RCC)*(A296+RCC)
      A299=(2.*A295*A296)**2-4.*A297*A298
      A300=SQRT(A299)
      A301=A300/A297
      A302=A301/A294
      A303=EXP(-ET1*A302)*A293
      A304=EXP(-ET2*A302)*A293
      A305=EXP(-ET3*A302)*A293
      A306=EXP(-ET4*A302)*A293
      A288=A288+A303
      A289=A289+A304
      A290=A290+A305
      A291=A291+A306
 316  CONTINUE
      E1F1=HY2*(A288+A204/2.)/A208
      E1F2=HY2*(A289+A204/2.)/A208
      E1F3=HY2*(A290+A204/2.)/A208
      E1F4=HY2*(A291+A204/2.)/A208
      A364=EXP(-EPT1*A207)*A204
      A365=EXP(-EPT2*A207)*A204
      A366=EXP(-EPT3*A207)*A204
      A367=EXP(-EPT4*A207)*A204
      A368=0.0
      A369=0.0
      A370=0.0
      A370=0.0
      DO 324 K38=1,NNK1
      A372=FLOAT(K38)*HY2
      A373=SIN(A372)
      A374=COS(A372)
      A375=A373/A374
      A376=RH*A375
      A377=A375**2+(RCC/RCL)**2
      A378=(A376-RCC)*(A376+RCC)
      A379=(2.*A375*A376)**2-4.*A377*A378
      A380=SQRT(A379)
      A381=(-2.*A375*A376+A380)/(2.*A377)
      A382=(RH+A381)/A374
      A383=RH*A374+SQRT(RP*RP-RH*RH*A373*A373)
      A384=A383-A382
      A385=EXP(-EPT1*A384)*A373
      A386=EXP(-EPT2*A384)*A373
      A387=EXP(-EPT3*A384)*A373
      A388=EXP(-EPT4*A384)*A373
      A368=A368+A385
      A369=A369+A386
      A370=A370+A387
      A371=A371+A388
 324  CONTINUE
      E2H1=HY2*(A368+A364/2.)/A208
      E2H2=HY2*(A369+A365/2.)/A208
      E2H3=HY2*(A370+A366/2.)/A208
      E2H4=HY2*(A371+A367/2.)/A208
      A400=EXP(-EPT1*A206)*A204
      A401=EXP(-EPT2*A206)*A204
      A402=EXP(-EPT3*A206)*A204
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      A403=EXP(-EPT4*A206)*A204
      A404=0.0
      A405=0.0
      A406=0.0
      A407=0.0
      HY3=(PI-A203)/FLOAT(NNK)
      DO 326 K39=1,NNK1
      A408=A203+FLOAT(K39)*HY3
      A409=SIN(A408)
      A410=COS(A408)
      A411=RH*A410+SQRT(RP*RP-RH*RH*A409*A409)
      A412=EXP(-EPT1*A411)*A409
      A413=EXP(-EPT2*A411)*A409
      A414=EXP(-EPT3*A411)*A409
      A415=EXP(-EPT4*A411)*A409
      A404=A404+A412
      A405=A405+A413
      A406=A406+A414
      A407=A407+A415
326   CONTINUE
      EHH1=HY3*(A404+A400/2.)/A209
      EHH2=HY3*(A405+A401/2.)/A209
      EHH3=HY3*(A406+A402/2.)/A209
      EHH4=HY3*(A407+A403/2.)/A209
      A500=-RCC*RCC/RH
      A501=SQRT(-A500*(RH+A500))*RCL/RCC
      A502=SQRT(A501*A501+(RH+A500)*(RH+A500))
      A503=ASIN(A501/A502)
      A504=SIN(A503)
      A505=COS(A503)
      A506=RH*A505+SQRT(RP*RP-RH*RH*A504*A504)
      A507=A506-A502
      A508=1.-A505
      A509=1.+A505
      GF3=A508/2.
      A510=EXP(-EPT1*A502)*A504
      A511=EXP(-EPT2*A502)*A504
      A512=EXP(-EPT3*A502)*A504
      A513=EXP(-EPT4*A502)*A504
      A515=0.0
      A516=0.0
      A517=0.0
      A518=0.0
      HY4=A503/FLOAT(NNK)
      DO 330 K40=1,NNK1
      A520=FLOAT(K40)*HY4
      A521=SIN(A520)
      A522=COS(A520)
      A523=A521/A522
      A524=RH*A523
      A525=A523**2+(RCL/RCC)**2
      A526=(A524-RCL)*(A524+RCL)
      A527=(2.*A523*A524)**2-4.*A525*A526
      A528=SQRT(A527)
      A529=(-2.*A523*A524-A528)/(2.*A525)
      A530=(RH+A529)/A522
      A531=EXP(-EPT1*A530)*A521
      A532=EXP(-EPT2*A530)*A521
      A533=EXP(-EPT3*A530)*A521
      A534=EXP(-EPT4*A530)*A521
      A515=A515+A531
      A516=A516+A532
      A517=A517+A533
      A518=A518+A534
330   CONTINUE
      V1H1=HY4*(A515+A510/2.)/A508
      V1H2=HY4*(A516+A511/2.)/A508
      V1H3=HY4*(A517+A512/2.)/A508
      V1H4=HY4*(A518+A513/2.)/A508
      A540=0.0
      A541=0.0
      A542=0.0
      A543=0.0
      DO 332 K41=1,NNK1
      A545=FLOAT(K41)*HY4
      A546=SIN(A545)
      A547=COS(A545)
      A548=A546/A547
      A549=RH*A548
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      A550=A548**2+(RCL/RCC)**2
      A551=(A549-RCL)*(A549+RCL)
      A552=(2.*A548*A549)**2-4.*A550*A551
      A553=SQRT(A552)
      A554=A553/A550
      A555=A554/A547
      A556=EXP(-ET1*A555)*A546
      A557=EXP(-ET2*A555)*A546
      A558=EXP(-ET3*A555)*A546
      A559=EXP(-ET4*A555)*A546
      A540=A540+A556
      A541=A541+A557
      A542=A542+A558
      A543=A543+A559
  332 CONTINUE
      V1F1=HY4*(A540+A504/2.)/A508
      V1F2=HY4*(A541+A504/2.)/A508
      V1F3=HY4*(A542+A504/2.)/A508
      V1F4=HY4*(A543+A504/2.)/A508
      A564=EXP(-EPT1*A507)*A504
      A565=EXP(-EPT2*A507)*A504
      A566=EXP(-EPT3*A507)*A504
      A567=EXP(-EPT4*A507)*A504
      A568=0.0
      A569=0.0
      A570=0.0
      A571=0.0
      DO 334 K42=1,NNK1
      A572=FLOAT(K42)*HY4
      A573=SIN(A572)
      A574=COS(A572)
      A575=A573/A574
      A576=RH*A575
      A577=A575**2+(RCL/RCC)**2
      A578=(A576-RCL)*(A576+RCL)
      A579=(2.*A575*A576)**2-4.*A577*A578
      A580=SQRT(A579)
      A581=(-2.*A575*A576+A580)/(2.*A577)
      A582=(RH+A581)/A574
      A583=RH*A574+SQRT(RP*RP-RH*RH*A573*A573)
      A584=A583-A582
      A585=EXP(-EPT1*A584)*A573
      A586=EXP(-EPT2*A584)*A573
      A587=EXP(-EPT3*A584)*A573
      A588=EXP(-EPT4*A584)*A573
      A568=A568+A585
      A569=A569+A586
      A570=A570+A587
      A571=A571+A588
  334 CONTINUE
      V2H1=HY4*(A568+A564/2.)/A508
      V2H2=HY4*(A569+A565/2.)/A508
      V2H3=HY4*(A570+A566/2.)/A508
      V2H4=HY4*(A571+A567/2.)/A508
      A600=EXP(-EPT1*A506)*A504
      A601=EXP(-EPT2*A506)*A504
      A602=EXP(-EPT3*A506)*A504
      A603=EXP(-EPT4*A506)*A504
      A604=0.0
      A605=0.0
      A606=0.0
      A607=0.0
      HY5=(PI-A503)/FLOAT(NNK)
      DO 336 K43=1,NNK1
      A608=A503+FLOAT(K43)*HY5
      A609=SIN(A608)
      A610=COS(A608)
      A611=RH*A610+SQRT(RP*RP-RH*RH*A609*A609)
      A612=EXP(-EPT1*A611)*A609
      A613=EXP(-EPT2*A611)*A609
      A614=EXP(-EPT3*A611)*A609
      A615=EXP(-EPT4*A611)*A609
      A604=A604+A612
      A605=A605+A613
      A606=A606+A614
      A607=A607+A615
  336 CONTINUE
      VHH1=HY5*(A604+A600/2.)/A509
      VHH2=HY5*(A605+A601/2.)/A509
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      VHH3=HY5*(A606+A602/2.)/A509
      VHH4=HY5*(A607+A603/2.)/A509
      A700=-RCC*RCC/RH
      A701=SQRT(-A700*(RH+A700))*RCS/RCC
      A702=SQRT(A701*A701+(RH+A700)*(RH+A700))
      A703=ASIN(A701/A702)
      A704=SIN(A703)
      A705=COS(A703)
      A706=RH*A705+SQRT(RP*RP-RH*RH*A704*A704)
      A707=A706-A702
      A708=1.-A705
      A709=1.+A705
      GF4=A708/2.
      A710=EXP(-EPT1*A702)*A704
      A711=EXP(-EPT2*A702)*A704
      A712=EXP(-EPT3*A702)*A704
      A713=EXP(-EPT4*A702)*A704
      A715=0.0
      A716=0.0
      A717=0.0
      A718=0.0
      HY6=A703/FLOAT(NNK)
      DO 338 K44=1,NNK1
      A720=FLOAT(K44)*HY6
      A721=SIN(A720)
      A722=COS(A720)
      A723=A721/A722
      A724=RH*A723
      A725=A723**2+(RCS/RCC)**2
      A726=(A724-RCS)*(A724+RCS)
      A727=(2.*A723*A724)**2-4.*A725*A726
      A728=SQRT(A727)
      A729=(-2.*A723*A724-A728)/(2.*A725)
      A730=(RH+A729)/A722
      A731=EXP(-EPT1*A730)*A721
      A732=EXP(-EPT2*A730)*A721
      A733=EXP(-EPT3*A730)*A721
      A734=EXP(-EPT4*A730)*A721
      A715=A715+A731
      A716=A716+A732
      A717=A717+A733
      A718=A718+A734
 338  CONTINUE
      F1H1=HY6*(A715+A710/2.)/A708
      F1H2=HY6*(A716+A711/2.)/A708
      F1H3=HY6*(A717+A712/2.)/A708
      F1H4=HY6*(A718+A713/2.)/A708
      A740=0.0
      A741=0.0
      A742=0.0
      A743=0.0
      DO 340 K45=1,NNK1
      A745=FLOAT(K45)*HY6
      A746=SIN(A745)
      A747=COS(A745)
      A748=A746/A747
      A749=RH*A748
      A750=A748**2+(RCS/RCC)**2
      A751=(A749-RCS)*(A749+RCS)
      A752=(2.*A748*A749)**2-4.*A750*A751
      A753=SQRT(A752)
      A754=A753/A750
      A755=A754/A747
      A756=EXP(-ET1*A755)*A746
      A757=EXP(-ET2*A755)*A746
      A758=EXP(-ET3*A755)*A746
      A759=EXP(-ET4*A755)*A746
      A740=A740+A756
      A741=A741+A757
      A742=A742+A758
      A743=A743+A759
 340  CONTINUE
      F1F1=HY6*(A740+A704/2.)/A708
      F1F2=HY6*(A741+A704/2.)/A708
      F1F3=HY6*(A742+A704/2.)/A708
      F1F4=HY6*(A743+A704/2.)/A708
      A764=EXP(-EPT1*A707)*A704
      A765=EXP(-EPT2*A707)*A704
      A766=EXP(-EPT3*A707)*A704
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      A767=EXP(-EPT4*A707)*A704
      A768=0.0
      A769=0.0
      A770=0.0
      A771=0.0
      DO 342 K46=1,NNK1
      A772=FLOAT(K46)*HY6
      A773=SIN(A772)
      A774=COS(A772)
      A775=A773/A774
      A776=RH*A775
      A777=A775**2+(RCS/RCC)**2
      A778=(A776-RCS)*(A776+RCS)
      A779=(2.*A775*A776)**2-4.*A777*A778
      A780=SQRT(A779)
      A781=(-2.*A775*A776+A780)/(2.*A777)
      A782=(RH+A781)/A774
      A783=RH*A774+SQRT(RP*RP-RH*RH*A773*A773)
      A784=A783-A782
      A785=EXP(-EPT1*A784)*A773
      A786=EXP(-EPT2*A784)*A773
      A787=EXP(-EPT3*A784)*A773
      A788=EXP(-EPT4*A784)*A773
      A768=A768+A785
      A769=A769+A786
      A770=A770+A787
      A771=A771+A788
342   CONTINUE
      F2H1=HY6*(A768+A764/2.)/A708
      F2H2=HY6*(A769+A765/2.)/A708
      F2H3=HY6*(A770+A766/2.)/A708
      F2H4=HY6*(A771+A767/2.)/A708
      A800=EXP(-EPT1*A706)*A704
      A801=EXP(-EPT2*A706)*A704
      A802=EXP(-EPT3*A706)*A704
      A803=EXP(-EPT4*A706)*A704
      A804=0.0
      A805=0.0
      A806=0.0
      A807=0.0
      HY7=(PI-A703)/FLOAT(NNK)
      DO 344 K47=1,NNK1
      A808=A703+FLOAT(K47)*HY7
      A809=SIN(A808)
      A810=COS(A808)
      A811=RH*A810+SQRT(RP*RP-RH*RH*A809*A809)
      A812=EXP(-EPT1*A811)*A809
      A813=EXP(-EPT2*A811)*A809
      A814=EXP(-EPT3*A811)*A809
      A815=EXP(-EPT4*A811)*A809
      A804=A804+A812
      A805=A805+A813
      A806=A806+A814
      A807=A807+A815
344   CONTINUE
      FHH1=HY7*(A804+A800/2.)/A709
      FHH2=HY7*(A805+A801/2.)/A709
      FHH3=HY7*(A806+A802/2.)/A709
      FHH4=HY7*(A807+A803/2.)/A709
      B20=RCC*COS(PI/8.)
      B21=RCC*SIN(PI/8.)
      B22=RCL*COS(PI/4.)
      B23=RCL*SIN(PI/4.)
      B24=RCC*COS(3.*PI/8.)
      B25=RCC*SIN(3.*PI/8.)
      B26=RCS
      B27=-RCC*COS(3.*PI/8.)
      B28=RCC*SIN(3.*PI/8.)
      B29=-RCL*COS(PI/4.)
      B30=RCL*SIN(PI/4.)
      B31=(RH+B20)**2+B21*B21
      B32=SQRT(B31)
      B33=(RH+B22)**2+B23*B23
      B34=SQRT(B33)
      B35=(RH+B24)**2+B25*B25
      B36=SQRT(B35)
      B37=SQRT(RH*RH+B26*B26)
      B38=(RH+B37)**2+B28*B28
      B39=SQRT(B38)
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      B40=(RH+B29)**2+B30*B30
      B41=SQRT(B40)
      B42=ASIN(B21/B32)
      B43=ASIN(B23/B34)
      B44=ASIN(B25/B36)
      B45=ASIN(B26/B37)
      B46=ASIN(B28/B39)
      B47=ASIN(B30/B41)
      B48=B21/B32
      B49=B23/B34
      B50=B25/B36
      B51=B26/B37
      B52=B28/B39
      B53=B30/B41
      B54=COS(B42)
      B55=COS(B43)
      B56=COS(B44)
      B57=COS(B45)
      B58=COS(B46)
      B59=COS(B47)
      B60=1.-B59
      B61=(RCC+RCS)/2.
      B62=(RCC+RCL)/2.
      B63=RH*B54-SQRT(B61*B61-RH*RH*B48*B48)
      B64=RH*B55-SQRT(B61*B61-RH*RH*B49*B49)
      B65=RH*B56-SQRT(B61*B61-RH*RH*B50*B50)
      B66=RH*B57-SQRT(B62*B62-RH*RH*B51*B51)
      B67=RH*B58-SQRT(B62*B62-RH*RH*B52*B52)
      B68=RH*B54+SQRT(RP*RP-RH*RH*B48*B48)
      B69=RH*B55+SQRT(RP*RP-RH*RH*B49*B49)
      B70=RH*B56+SQRT(RP*RP-RH*RH*B50*B50)
      B71=RH*B57+SQRT(RP*RP-RH*RH*B51*B51)
      B72=RH*B58+SQRT(RP*RP-RH*RH*B52*B52)
      B73=RH*B59+SQRT(RP*RP-RH*RH*B53*B53)
      B75=EXP(-EPT1*B63)*B48
      B76=EXP(-EPT1*B64)*B49
      B77=EXP(-EPT1*B65)*B50
      B78=EXP(-EPT1*B66)*B51
      B79=EXP(-EPT1*B67)*B52
      B80=EXP(-EPT1*B41)*B53
      B81=EXP(-EPT2*B63)*B48
      B82=EXP(-EPT2*B64)*B49
      B83=EXP(-EPT2*B65)*B50
      B84=EXP(-EPT2*B66)*B51
      B85=EXP(-EPT2*B67)*B52
      B86=EXP(-EPT2*B41)*B53
      B87=EXP(-EPT3*B63)*B48
      B88=EXP(-EPT3*B64)*B49
      B89=EXP(-EPT3*B65)*B50
      B90=EXP(-EPT3*B66)*B51
      B91=EXP(-EPT3*B67)*B52
      B92=EXP(-EPT3*B41)*B53
      B93=EXP(-EPT4*B63)*B48
      B94=EXP(-EPT4*B64)*B49
      B95=EXP(-EPT4*B65)*B50
      B96=EXP(-EPT4*B66)*B51
      B97=EXP(-EPT4*B67)*B52
      B98=EXP(-EPT4*B41)*B53
      B100=(B75*B42+(B75+B76)*(B43-B42)+(B76+B77)*(B44-B43)+(
     #B77+B78)*(B45-B44)+(B78+B79)*(B46-B45)+(B79+B80)*(B47-
     #B46))/2.
      Z1H1=B100/B60
      B101=(B81*B42+(B81+B82)*(B43-B42)+(B82+B83)*(B44-B43)+(
     #B83+B84)*(B45-B44)+(B84+B85)*(B46-B45)+(B85+B86)*(B47-
     #B46))/2.
      Z1H2=B101/B60
      B102=(B87*B42+(B87+B88)*(B43-B42)+(B88+B89)*(B44-B43)+(
     #B89+B90)*(B45-B44)+(B90+B91)*(B46-B45)+(B91+B92)*(B47-
     #B46))/2.
      Z1H3=B102/B60
      B103=(B93*B42+(B93+B94)*(B43-B42)+(B94+B95)*(B44-B43)+(
     #B95+B96)*(B45-B44)+(B96+B97)*(B46-B45)+(B97+B98)*(B47-
     #B46))/2.
      Z1H4=B103/B60
      GF5=B60/2.
      B105=B32-B63
      B106=B34-B64
      B107=B36-B65
      B108=B37-B66
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 B109=B39-B67
 B110=EXP(-ET1*B105)*B48
 B111=EXP(-ET1*B106)*B49
 B112=EXP(-ET1*B107)*B50
 B113=EXP(-ET1*B108)*B51
 B114=EXP(-ET1*B109)*B52
 B115=EXP(-ET2*B105)*B48
 B116=EXP(-ET2*B106)*B49
 B117=EXP(-ET2*B107)*B50
 B118=EXP(-ET2*B108)*B51
 B119=EXP(-ET2*B109)*B52
 B120=EXP(-ET3*B105)*B48
 B121=EXP(-ET3*B106)*B49
 B122=EXP(-ET3*B107)*B50
 B123=EXP(-ET3*B108)*B51
 B124=EXP(-ET3*B109)*B52
 B125=EXP(-ET4*B105)*B48
 B126=EXP(-ET4*B106)*B49
 B127=EXP(-ET4*B107)*B50
 B128=EXP(-ET4*B108)*B51
 B129=EXP(-ET4*B109)*B52
 B130=(B110*B42+(B110+B111)*(B43-B42)+(B111+B112)*(B44-B43
#)+(B112+B113)*(B45-B44)+(B113+B114)*(B46-B45)+(B114+B53)*
#(B47-B46))/2.
 Z1F1=B130/B60
 B131=(B115*B42+(B115+B116)*(B43-B42)+(B116+B117)*(B44-
#B43)+(B117+B118)*(B45-B44)+(B118+B119)*(B46-B45)+
#(B119+B53)*(B47-B46))/2.
 Z1F2=B131/B60
 B132=(B120*B42+(B120+B121)*(B43-B42)+(B121+B122)*(B44-
#B43)+(B122+B123)*(B45-B44)+(B123+B124)*(B46-B45)+(
#B124+B53)*(B47-B46))/2.
 Z1F3=B132/B60
 B133=(B125*B42+(B125+B126)*(B43-B42)+(B126+B127)*(B44-
#B43)+(B127+B128)*(B45-B44)+(B128+B129)*(B46-B45)+(
#B129+B53)*(B47-B46))/2.
 Z1F4=B133/B60
 B135=B68-B32
 B136=B69-B34
 B137=B70-B36
 B138=B71-B37
 B139=B72-B39
 B140=B73-B41
 B141=EXP(-EPT1*B135)*B48
 B142=EXP(-EPT1*B136)*B49
 B143=EXP(-EPT1*B137)*B50
 B144=EXP(-EPT1*B138)*B51
 B145=EXP(-EPT1*B139)*B52
 B146=EXP(-EPT1*B140)*B53
 B147=EXP(-EPT2*B135)*B48
 B148=EXP(-EPT2*B136)*B49
 B149=EXP(-EPT2*B137)*B50
 B150=EXP(-EPT2*B138)*B51
 B151=EXP(-EPT2*B139)*B52
 B152=EXP(-EPT2*B140)*B53
 B154=EXP(-EPT3*B135)*B48
 B155=EXP(-EPT3*B136)*B49
 B156=EXP(-EPT3*B137)*B50
 B157=EXP(-EPT3*B138)*B51
 B158=EXP(-EPT3*B139)*B52
 B159=EXP(-EPT3*B140)*B53
 B160=EXP(-EPT4*B135)*B48
 B161=EXP(-EPT4*B136)*B49
 B162=EXP(-EPT4*B137)*B50
 B163=EXP(-EPT4*B138)*B51
 B164=EXP(-EPT4*B139)*B52
 B165=EXP(-EPT4*B140)*B53
 B166=(B141*B42+(B141+B142)*(B43-B42)+(B142+B143)*(B44-
#B43)+(B143+B144)*(B45-B44)+(B144+B145)*(B46-B45)+(
#B145+B146)*(B47-B46))/2.
 Z2H1=B166/B60
 B167=(B147*B42+(B147+B148)*(B43-B42)+(B148+B149)*(B44-
#B43)+(B149+B150)*(B45-B44)+(B150+B151)*(B46-B45)+(
#B151+B152)*(B47-B46))/2.
 Z2H2=B167/B60
 B168=(B154*B42+(B154+B155)*(B43-B42)+(B155+B156)*(B44-
#B43)+(B156+B157)*(B45-B44)+(B157+B158)*(B46-B45)+(
#B158+B159)*(B47-B46))/2.
 Z2H3=B168/B60
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      B169=(B160*B42+(B160+B161)*(B43-B42)+(B161+B162)*(B44-
     #B43)+(B162+B163)*(B45-B44)+(B163+B164)*(B46-B45)+(
     #B164+B165)*(B47-B46))/2.
      Z2H4=B169/B60
      B170=1.+B59
      HJ1=(PI-B47)/FLOAT(NNK)
      B171=EXP(-EPT1*B73)*B53
      B172=EXP(-EPT2*B73)*B53
      B173=EXP(-EPT3*B73)*B53
      B174=EXP(-EPT4*B73)*B53
      B175=0.0
      B176=0.0
      B177=0.0
      B178=0.0
      DO 346 K50=1,NNK1
      B179=B47+FLOAT(K50)*HJ1
      B180=SIN(B179)
      B181=COS(B179)
      B182=RH*B181+SQRT(RP*RP-RH*RH*B180*B180)
      B183=EXP(-EPT1*B182)*B180
      B184=EXP(-EPT2*B182)*B180
      B185=EXP(-EPT3*B182)*B180
      B186=EXP(-EPT4*B182)*B180
      B175=B175+B183
      B176=B176+B184
      B177=B177+B185
      B178=B178+B186
346   CONTINUE
      ZHH1=HJ1*(B175+B171/2.)/B170
      ZHH2=HJ1*(B176+B172/2.)/B170
      ZHH3=HJ1*(B177+B173/2.)/B170
      ZHH4=HJ1*(B178+B174/2.)/B170
      B200=-RCS*RCS/RH
      B201=SQRT(-B200*(RH+B200))*RCC/RCS
      B202=(RH+B200)**2+B201*B201
      B203=SQRT(B202)
      B204=B201/B203
      B205=(RH+B200)/B203
      B206=ASIN(B204)
      B207=1.-B205
      B208=1.+B205
      GF6=B207/2.
      B209=RH*B205+SQRT(RP*RP-RH*RH*B204*B204)
      B210=B209-B203
      B211=EXP(-EPT1*B203)*B204
      B212=EXP(-EPT2*B203)*B204
      B213=EXP(-EPT3*B203)*B204
      B214=EXP(-EPT4*B203)*B204
      B215=0.0
      B216=0.0
      B217=0.0
      B218=0.0
      B220=0.0
      B221=0.0
      B222=0.0
      B223=0.0
      B225=EXP(-EPT1*B210)*B204
      B226=EXP(-EPT2*B210)*B204
      B227=EXP(-EPT3*B210)*B204
      B228=EXP(-EPT4*B210)*B204
      B229=0.0
      B230=0.0
      B231=0.0
      B232=0.0
      B240=EXP(-EPT1*B209)*B204
      B241=EXP(-EPT2*B209)*B204
      B242=EXP(-EPT3*B209)*B204
      B243=EXP(-EPT4*B209)*B204
      B244=0.0
      B245=0.0
      B246=0.0
      B247=0.0
      HJ2=B206/FLOAT(NNK)
      DO 348 K51=1,NNK1
      B300=FLOAT(K51)*HJ2
      B301=SIN(B300)
      B302=COS(B300)
      B303=B301/B302
      B304=RH*B303
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      B305=B303**2+(RCC/RCS)**2
      B306=(B304+RCC)*(B304-RCC)
      B307=(2.*B303*B304)**2-4.*B305*B306
      B308=SQRT(B307)
      B309=(-2.*B303*B304-B308)/(2.*B305)
      B310=(RH+B309)/B302
      B311=(-2.*B303*B304+B308)/(2.*B305)
      B312=(RH+B311)/B302
      B313=RH*B302+SQRT(RP*RP-RH*RH*B301*B301)
      B315=EXP(-EPT1*B310)*B301
      B316=EXP(-EPT2*B310)*B301
      B317=EXP(-EPT3*B310)*B301
      B318=EXP(-EPT4*B310)*B301
      B215=B215+B315
      B216=B216+B316
      B217=B217+B317
      B218=B218+B318
      B320=B312-B310
      B321=EXP(-ET1*B320)*B301
      B322=EXP(-ET2*B320)*B301
      B323=EXP(-ET3*B320)*B301
      B324=EXP(-ET4*B320)*B301
      B220=B220+B321
      B221=B221+B322
      B222=B222+B323
      B223=B223+B324
      B330=B313-B312
      B331=EXP(-EPT1*B330)*B301
      B332=EXP(-EPT2*B330)*B301
      B333=EXP(-EPT3*B330)*B301
      B334=EXP(-EPT4*B330)*B301
      B229=B229+B331
      B230=B230+B332
      B231=B231+B333
      B232=B232+B334
348   CONTINUE
      G1H1=HJ2*(B215+B211/2.)/B207
      G1H2=HJ2*(B216+B212/2.)/B207
      G1H3=HJ2*(B217+B213/2.)/B207
      G1H4=HJ2*(B218+B214/2.)/B207
      G1F1=HJ2*(B220+B204/2.)/B207
      G1F2=HJ2*(B221+B204/2.)/B207
      G1F3=HJ2*(B222+B204/2.)/B207
      G1F4=HJ2*(B223+B204/2.)/B207
      G2H1=HJ2*(B229+B225/2.)/B207
      G2H2=HJ2*(B230+B226/2.)/B207
      G2H3=HJ2*(B231+B227/2.)/B207
      G2H4=HJ2*(B232+B228/2.)/B207
      HJ3=(PI-B206)/FLOAT(NNK)
      DO 350 K52=1,NNK1
      B340=B206+FLOAT(K52)*HJ3
      B341=SIN(B340)
      B342=COS(B340)
      B343=RH*B342+SQRT(RP*RP-RH*RH*B341*B341)
      B344=EXP(-EPT1*B343)*B341
      B345=EXP(-EPT2*B343)*B341
      B346=EXP(-EPT3*B343)*B341
      B347=EXP(-EPT4*B343)*B341
      B244=B244+B344
      B245=B245+B345
      B246=B246+B346
      B247=B247+B347
350   CONTINUE
      GHH1=HJ3*(B244+B240/2.)/B208
      GHH2=HJ3*(B245+B241/2.)/B208
      GHH3=HJ3*(B246+B242/2.)/B208
      GHH4=HJ3*(B247+B243/2.)/B208
      GCC=(GF1+GF2+GF5+GF6)*0.2+(GF3+GF4)*0.1
      GPP=1.-GCC
      HYP11=(T1H1+E1H1+Z1H1+G1H1)*0.2+(V1H1+F1H1)*0.1
      HYP12=(T1H2+E1H2+Z1H2+G1H2)*0.2+(V1H2+F1H2)*0.1
      HYP13=(T1H3+E1H3+Z1H3+G1H3)*0.2+(V1H3+F1H3)*0.1
      HYP14=(T1H4+E1H4+Z1H4+G1H4)*0.2+(V1H4+F1H4)*0.1
      HYCC1=(T1F1+E1F1+Z1F1+G1F1)*0.2+(V1F1+F1F1)*0.1
      HYCC2=(T1F2+E1F2+Z1F2+G1F2)*0.2+(V1F2+F1F2)*0.1
      HYCC3=(T1F3+E1F3+Z1F3+G1F3)*0.2+(V1F3+F1F3)*0.1
      HYCC4=(T1F4+E1F4+Z1F4+G1F4)*0.2+(V1F4+F1F4)*0.1
      HYP21=(T2H1+E2H1+Z2H1+G2H1)*0.2+(V2H1+F2H1)*0.1
      HYP22=(T2H2+E2H2+Z2H2+G2H2)*0.2+(V2H2+F2H2)*0.1
```

```
      HYP23=(T2H3+E2H3+Z2H3+G2H3)*0.2+(V2H3+F2H3)*0.1
      HYP24=(T2H4+E2H4+Z2H4+G2H4)*0.2+(V2H4+F2H4)*0.1
      HXPP1=(THH1+EHH1+ZHH1+GHH1)*0.2+(VHH1+FHH1)*0.1
      HXPP2=(THH2+EHH2+ZHH2+GHH2)*0.2+(VHH2+FHH2)*0.1
      HXPP3=(THH3+EHH3+ZHH3+GHH3)*0.2+(VHH3+FHH3)*0.1
      HXPP4=(THH4+EHH4+ZHH4+GHH4)*0.2+(VHH4+FHH4)*0.1
      GC1=GCC
      GC2=GCC
      GC3=GCC
      GC4=GCC
      GP1=GPP
      GP2=GPP
      GP3=GPP
      GP4=GPP
      YP11=HYP11
      YP12=HYP12
      YP13=HYP13
      YP14=HYP14
      YCC1=HYCC1
      YCC2=HYCC2
      YCC3=HYCC3
      YCC4=HYCC4
      YP21=HYP21
      YP22=HYP22
      YP23=HYP23
      YP24=HYP24
      XPP1=HXPP1
      XPP2=HXPP2
      XPP3=HXPP3
      XPP4=HXPP4
 500  CONTINUE
C
C     **********************************************************
C     NEUTRON ABSORPTION BY THE FUEL AND HYDROGEN REGIONS
C     **********************************************************
C
      H100=SAC*EPP4*(1.-EF4)/H11
      H101=H100
      H103=(SAP*(1.-OPP4)+SAC*(1.-EPP4)+SAC*EPP4*EF4*(1.-EPP4))/
     #H11
      H104=H103*EPA4/EPT4
      H105=H103*EPS44/EPT4
      H106=1.-TC4
      H107=H106
      H109=TC4*(1.-TP4)
      H110=H109*EPA4/EPT4
      H111=H109*EPS44/EPT4
      H112=TC4*TP4*RA44
      H113=H107+H112*H101
      H115=H110+H112*H104
      H116=H111+H112*H105
      H117=(GP4*XPP4+GC4*YP14*YCC4*YP24)*RA44
      H118=GC4*YP14*(1.-YCC4)
      H119=H118
      H121=GP4*(1.-XPP4)+GC4*(1.-YP14)+GC4*YP14*YCC4*(1.-YP24)
      H122=H121*EPA4/EPT4
      H123=H121*EPS44/EPT4
      H124=H119+H117*H101
      H126=H122+H117*H104
      H127=H123+H117*H105
      H128=H124/(1.-H127)
      H130=H126/(1.-H127)
      H131=H113+H116*H128
      H133=H115+H116*H130
      Q11=H131
      Q10=H133
      Q9=H128
      Q8=H130
      Q12=H104+H105*Q8
      Q13=H101+H105*Q9
      Q20=(SAC*EPP3*EPP3*EF3+SAP*OPP3)*RA34/H12
      QQ20=SAC*EPP3*(1.-EF3)/H12
      Q21=QQ20*EA3/ET3
      Q22=QQ20*ES33/ET3
      Q23=QQ20*ES34/ET3
      Q24=(SAP*(1.-OPP3)+SAC*(1.-EPP3)+SAC*EPP3*EF3*(1.-EPP3))/
     #H12
      Q25=Q24*EPA3/EPT3
      Q26=Q24*EPS33/EPT3
```

```
Q27=Q24*EPS34/EPT3
Q28=(GP3*XPP3+GC3*YP13*YCC3*YP23)*RA34
Q29=(GP3*XPP3+GC3*YP13*YCC3*YP23)*RA33
QQ29=GC3*YP13*(1.-YCC3)
Q30=QQ29*EA3/ET3
Q31=QQ29*ES33/ET3
Q32=QQ29*ES34/ET3
Q33=GP3*(1.-XPP3)+GC3*(1.-YP13)+GC3*YP13*YCC3*(1.-YP23)
Q34=Q33*EPA3/EPT3
Q35=Q33*EPS33/EPT3
Q36=Q33*EPS34/EPT3
Q37=Q20*Q13+Q23*Q11+Q27*Q9
Q38=Q20*Q12+Q23*Q10+Q27*Q8
Q39=Q28*Q13+Q32*Q11+Q36*Q9+Q29*Q37
Q40=Q28*Q12+Q32*Q10+Q36*Q8+Q29*Q38
Q42=Q30+Q29*Q21
Q43=Q34+Q29*Q25
Q44=Q35+Q29*Q26
Q45=Q31+Q29*Q22
Q46=TC3*TP3*RA34
Q47=TC3*TP3*RA33
QQ47=1.-TC3
Q48=QQ47*EA3/ET3
Q49=QQ47*ES33/ET3
Q50=QQ47*ES34/ET3
QQ50=TC3*(1.-TP3)
Q51=QQ50*EPA3/EPT3
Q52=QQ50*EPS33/EPT3
Q53=QQ50*EPS34/EPT3
Q54=Q46*Q13+Q50*Q11+Q53*Q9+Q47*Q37
Q55=Q46*Q12+Q50*Q10+Q53*Q8+Q47*Q38
Q56=Q48+Q47*Q21
Q57=Q51+Q47*Q25
Q58=Q49+Q47*Q22
Q59=Q52+Q47*Q26
Q390=1.-Q44
Q60=Q54+Q59*Q39/Q390
Q61=Q55+Q59*Q40/Q390
Q62=Q56+Q59*Q42/Q390
Q63=Q57+Q59*Q43/Q390
Q64=Q58+Q59*Q45/Q390
Q391=1.-Q64
Q65=Q60/Q391
Q66=Q61/Q391
Q67=Q62/Q391
Q68=Q63/Q391
Q69=(Q39+Q45*Q65)/Q390
Q70=(Q40+Q45*Q66)/Q390
Q71=(Q42+Q45*Q67)/Q390
Q72=(Q43+Q45*Q68)/Q390
Q73=Q37+Q22*Q65+Q26*Q69
Q74=Q38+Q22*Q66+Q26*Q70
Q75=Q21+Q22*Q67+Q26*Q71
Q76=Q25+Q22*Q68+Q26*Q72
Q100=(SAC*EPP2*EPP2*EF2+SAP*OPP2)*RA24/H13
Q101=(SAC*EPP2*EPP2*EF2+SAP*OPP2)*RA23/H13
QQ101=SAC*EPP2*(1.-EF2)/H13
Q102=QQ101*EA2/ET2
Q103=QQ101*ES22/ET2
Q104=QQ101*ES23/ET2
Q105=QQ101*ES24/ET2
Q106=(SAP*(1.-OPP2)+SAC*(1.-EPP2)+SAC*EPP2*EF2*(1.-EPP2))/
#H13
Q107=Q106*EPA2/EPT2
Q108=Q106*EPS22/EPT2
Q109=Q106*EPS23/EPT2
Q110=Q106*EPS24/EPT2
Q111=Q100*Q13+Q105*Q11+Q110*Q9+Q101*Q73+Q104*Q65+Q109*Q69
Q112=Q100*Q12+Q105*Q10+Q110*Q8+Q101*Q74+Q104*Q66+Q109*Q70
Q113=Q101*Q75+Q104*Q67+Q109*Q71
Q114=Q101*Q76+Q104*Q68+Q109*Q72
Q115=TC2*TP2*RA24
Q116=TC2*TP2*RA23
Q117=TC2*TP2*RA22
QQ117=1.-TC2
Q118=QQ117*EA2/ET2
Q119=QQ117*ES22/ET2
Q120=QQ117*ES23/ET2
Q121=QQ117*ES24/ET2
```

```
 QQ122=TC2*(1.-TP2)
 Q122=QQ122*EPA2/EPT2
 Q123=QQ122*EPS22/EPT2
 Q124=QQ122*EPS23/EPT2
 Q125=QQ122*EPS24/EPT2
 Q126=(GP2*XPP2+GC2*YP12*YCC2*YP22)*RA24
 Q127=(GP2*XPP2+GC2*YP12*YCC2*YP22)*RA23
 Q128=(GP2*XPP2+GC2*YP12*YCC2*YP22)*RA22
 QQ129=GC2*YP12*(1.-YCC2)
 Q129=QQ129*EA2/ET2
 Q130=QQ129*ES22/ET2
 Q131=QQ129*ES23/ET2
 Q132=QQ129*ES24/ET2
 Q133=GP2*(1.-XPP2)+GC2*(1.-YP12)+GC2*YP12*YCC2*(1.-YP22)
 Q134=Q133*EPA2/EPT2
 Q135=Q133*EPS22/EPT2
 Q136=Q133*EPS23/EPT2
 Q137=Q133*EPS24/EPT2
 Q138=Q126*Q13+Q132*Q11+Q137*Q9+Q127*Q73+Q131*Q65+Q136*Q69+
#Q128*Q111
 Q139=Q126*Q12+Q132*Q10+Q137*Q8+Q127*Q74+Q131*Q66+Q136*Q70+
#Q128*Q112
 Q140=Q127*Q75+Q131*Q67+Q136*Q71+Q128*Q113
 Q141=Q127*Q76+Q131*Q68+Q136*Q72+Q128*Q114
 Q142=Q129+Q128*Q102
 Q143=Q130+Q128*Q103
 Q144=Q134+Q128*Q107
 Q145=Q135+Q128*Q108
 Q146=Q115*Q13+Q121*Q11+Q125*Q9+Q116*Q73+Q120*Q65+Q124*Q69+
#Q117*Q111
 Q147=Q115*Q12+Q121*Q10+Q125*Q8+Q116*Q74+Q120*Q66+Q124*Q70+
#Q117*Q112
 Q148=Q116*Q75+Q120*Q67+Q124*Q71+Q117*Q113
 Q149=Q116*Q76+Q120*Q68+Q124*Q72+Q117*Q114
 Q392=(Q123+Q117*Q108)/(1.-Q145)
 Q150=Q146+Q138*Q392
 Q151=Q147+Q139*Q392
 Q152=Q148+Q140*Q392
 Q153=Q149+Q141*Q392
 Q154=Q118+Q117*Q102+Q142*Q392
 Q155=Q122+Q117*Q107+Q144*Q392
 Q156=Q119+Q117*Q103+Q143*Q392
 Q393=1.-Q156
 Q157=Q150/Q393
 Q158=Q151/Q393
 Q159=Q152/Q393
 Q160=Q153/Q393
 Q161=Q154/Q393
 Q162=Q155/Q393
 Q163=Q143/(1.-Q145)
 Q394=1.-Q145
 Q164=Q138/Q394+Q163*Q157
 Q165=Q139/Q394+Q163*Q158
 Q166=Q140/Q394+Q163*Q159
 Q167=Q141/Q394+Q163*Q160
 Q168=Q142/Q394+Q163*Q161
 Q169=Q144/Q394+Q163*Q162
 Q170=Q111+Q103*Q157+Q108*Q164
 Q171=Q112+Q103*Q158+Q108*Q165
 Q172=Q113+Q103*Q159+Q108*Q166
 Q173=Q114+Q103*Q160+Q108*Q167
 Q174=Q102+Q103*Q161+Q108*Q168
 Q175=Q107+Q103*Q162+Q108*Q169
 Q190=SAC*EPP1*EPP1*EF1+SAP*OPP1
 Q200=Q190*RA14/H14
 Q201=Q190*RA13/H14
 Q202=Q190*RA12/H14
 QQ203=SAC*EPP1*(1.-EF1)/H14
 Q203=QQ203*EA1/ET1
 Q204=QQ203*ES11/ET1
 Q205=QQ203*ES12/ET1
 Q206=QQ203*ES13/ET1
 Q207=QQ203*ES14/ET1
 Q208=(SAP*(1.-OPP1)+SAC*(1.-EPP1)+SAC*EPP1*EF1*(1.-EPP1))/
#H14
 Q209=Q208*EPA1/EPT1
 Q210=Q208*EPS11/EPT1
 Q211=Q208*EPS12/EPT1
 Q212=Q208*EPS13/EPT1
```

```
 Q213=Q208*EPS14/EPT1
 Q214=Q200*Q13+Q207*Q11+Q213*Q9+Q201*Q73+Q206*Q65+Q212*Q69+
#Q202*Q170+Q205*Q157+Q211*Q164
 Q215=Q200*Q12+Q207*Q10+Q213*Q8+Q201*Q74+Q206*Q66+Q212*Q70+
#Q202*Q171+Q205*Q158+Q211*Q165
 Q216=Q201*Q75+Q206*Q67+Q212*Q71+Q202*Q172+Q205*Q159+Q211*
#Q166
 Q217=Q201*Q76+Q206*Q68+Q212*Q72+Q202*Q173+Q205*Q160+Q211*
#Q167
 Q218=Q202*Q174+Q205*Q161+Q211*Q168
 Q219=Q202*Q175+Q205*Q162+Q211*Q169
 Q220=TC1*TP1*RA14
 Q221=TC1*TP1*RA13
 Q222=TC1*TP1*RA12
 Q223=TC1*TP1*RA11
 QQ224=1.-TC1
 Q224=QQ224*EA1/ET1
 Q225=QQ224*ES11/ET1
 Q226=QQ224*ES12/ET1
 Q227=QQ224*ES13/ET1
 Q228=QQ224*ES14/ET1
 QQ229=TC1*(1.-TP1)
 Q229=QQ229*EPA1/EPT1
 Q230=QQ229*EPS11/EPT1
 Q231=QQ229*EPS12/EPT1
 Q232=QQ229*EPS13/EPT1
 Q233=QQ229*EPS14/EPT1
 Q234=Q220*Q13+Q228*Q11+Q233*Q9+Q221*Q73+Q227*Q65+Q232*Q69+
#Q222*Q170+Q226*Q157+Q231*Q164+Q223*Q214
 Q235=Q220*Q12+Q228*Q10+Q233*Q8+Q221*Q74+Q227*Q66+Q232*Q70+
#Q222*Q171+Q226*Q158+Q231*Q165+Q223*Q215
 Q236=Q221*Q75+Q227*Q67+Q232*Q71+Q222*Q172+Q226*Q159+Q231*
#Q166+Q223*Q216
 Q237=Q221*Q76+Q227*Q68+Q232*Q72+Q222*Q173+Q226*Q160+Q231*
#Q167+Q223*Q217
 Q238=Q222*Q174+Q226*Q161+Q231*Q168+Q223*Q218
 Q239=Q222*Q175+Q226*Q162+Q231*Q169+Q223*Q219
 Q240=Q223*Q203+Q224
 Q241=Q223*Q204+Q225
 Q242=Q223*Q209+Q229
 Q243=Q223*Q210+Q230
 Q191=GP1*XPP1+GC1*YP11*YCC1*YP21
 Q244=Q191*RA14
 Q245=Q191*RA13
 Q246=Q191*RA12
 Q247=Q191*RA11
 QQ248=GC1*YP11*(1.-YCC1)
 Q248=QQ248*EA1/ET1
 Q249=QQ248*ES11/ET1
 Q250=QQ248*ES12/ET1
 Q251=QQ248*ES13/ET1
 Q252=QQ248*ES14/ET1
 Q253=GP1*(1.-XPP1)+GC1*(1.-YP11)+GC1*YP11*YCC1*(1.-YP21)
 Q254=Q253*EPA1/EPT1
 Q255=Q253*EPS11/EPT1
 Q256=Q253*EPS12/EPT1
 Q257=Q253*EPS13/EPT1
 Q258=Q253*EPS14/EPT1
 Q259=Q244*Q13+Q252*Q11+Q258*Q9+Q245*Q73+Q251*Q65+Q257*Q69+
#Q246*Q170+Q250*Q157+Q256*Q164+Q247*Q214
 Q260=Q244*Q12+Q252*Q10+Q258*Q8+Q245*Q74+Q251*Q66+Q257*Q70+
#Q246*Q171+Q250*Q158+Q256*Q165+Q247*Q215
 Q261=Q245*Q75+Q251*Q67+Q257*Q71+Q246*Q172+Q250*Q159+Q256*
#Q166+Q247*Q216
 Q262=Q245*Q76+Q251*Q68+Q257*Q72+Q246*Q173+Q250*Q160+Q256*
#Q167+Q247*Q217
 Q263=Q246*Q174+Q250*Q161+Q256*Q168+Q247*Q218
 Q264=Q246*Q175+Q250*Q162+Q256*Q169+Q247*Q219
 Q265=Q247*Q203+Q248
 Q266=Q247*Q204+Q249
 Q267=Q247*Q209+Q254
 Q268=Q247*Q210+Q255
 Q400=1.-Q268
 Q269=Q259/Q400
 Q270=Q260/Q400
 Q271=Q261/Q400
 Q272=Q262/Q400
 Q273=Q263/Q400
 Q274=Q264/Q400
```

```
Q275=Q265/Q400
Q276=Q266/Q400
Q277=Q267/Q400
Q278=Q234+Q243*Q269
Q279=Q235+Q243*Q270
Q280=Q236+Q243*Q271
Q281=Q237+Q243*Q272
Q282=Q238+Q243*Q273
Q283=Q239+Q243*Q274
Q284=Q240+Q243*Q275
Q285=Q241+Q243*Q276
Q286=Q242+Q243*Q277
Q401=1.-Q285
Q287=Q278/Q401
Q288=Q279/Q401
Q289=Q280/Q401
Q290=Q281/Q401
Q291=Q282/Q401
Q292=Q283/Q401
Q293=Q284/Q401
Q294=Q286/Q401
Q295=Q269+Q276*Q287
Q296=Q270+Q276*Q288
Q297=Q271+Q276*Q289
Q298=Q272+Q276*Q290
Q299=Q273+Q276*Q291
Q300=Q274+Q276*Q292
Q301=Q275+Q276*Q293
Q302=Q277+Q276*Q294
Q303=Q214+Q204*Q287+Q210*Q295
Q304=Q215+Q204*Q288+Q210*Q296
Q305=Q216+Q204*Q289+Q210*Q297
Q306=Q217+Q204*Q290+Q210*Q298
Q307=Q218+Q204*Q291+Q210*Q299
Q308=Q219+Q204*Q292+Q210*Q300
Q309=Q203+Q204*Q293+Q210*Q301
Q310=Q209+Q204*Q294+Q210*Q302
Q500=(1.-TC4)*EA4/ET4
Q501=(1.-TC4)*ES44/ET4
Q502=Q500/(1.-Q501)
Q503=(1.-TC3)*EA3/ET3
Q504=(1.-TC3)*ES33/ET3
Q505=(1.-TC3)*ES34/ET3
Q506=Q503/(1.-Q504)
Q507=Q505/(1.-Q504)
Q508=Q507*Q502
Q509=(1.-TC2)*EA2/ET2
Q510=(1.-TC2)*ES22/ET2
Q511=(1.-TC2)*ES23/ET2
Q512=(1.-TC2)*ES24/ET2
Q513=Q509/(1.-Q510)
Q514=Q511/(1.-Q510)
Q515=Q512/(1.-Q510)
Q516=Q515*Q502+Q514*Q508
Q517=Q514*Q506
Q518=(1.-TC1)*EA1/ET1
Q519=(1.-TC1)*ES11/ET1
Q520=(1.-TC1)*ES12/ET1
Q521=(1.-TC1)*ES13/ET1
Q522=(1.-TC1)*ES14/ET1
Q523=Q518/(1.-Q519)
Q524=Q520/(1.-Q519)
Q525=Q521/(1.-Q519)
Q526=Q522/(1.-Q519)
Q527=Q524*Q513
Q528=Q524*Q517+Q525*Q506
Q529=Q524*Q516+Q525*Q508+Q526*Q502
Q530=CHI1*Q523
Q531=CHI1*Q527+CHI2*Q513
Q532=CHI1*Q528+CHI2*Q517
Q533=CHI1*Q529+CHI2*Q516
Q320=CHI1*Q293
Q321=CHI1*Q291+CHI2*Q161
Q322=CHI1*Q289+CHI2*Q159
Q323=CHI1*Q287+CHI2*Q157
Q534=Q320-Q530
Q535=Q321-Q531
Q536=Q322-Q532
Q537=Q323-Q533
```

```
      Q324=Q320+Q321+Q322+Q323
      Q538=Q530+Q531+Q531+Q533
      Q539=Q324-Q538
      Q330=CHI1*Q294
      Q331=CHI1*Q292+CHI2*Q162
      Q332=CHI1*Q290+CHI2*Q160
      Q333=CHI1*Q288+CHI2*Q158
      Q334=Q330+Q331+Q332+Q333
C
C     *******************************************************
C     EFFECTIVE NEUTRON MULTIPLICATION FACTOR CALCULATION
C     *******************************************************
C
      EKEF=Q320*(FV1+2.*EN1)/EA1+Q321*FV2/EA2+Q322*FV3/EA3+
     #Q323*FV4/EA4

C
C     *******************************************************
C                           OUTPUT RESULTS
C
C     REFLECTOR ALBEDOS ( SEE TABLE 7-3 )
C
C     RA11 = PROBABILITY THAT A NEUTRON FROM GROUP 1 BE REFLECTED
C            WITH GROUP 1 ENERGY
C     RA12 = PROBABILITY THAT A NEUTRON FROM GROUP 1 BE REFLECTED
C            WITH GROUP 2 ENERGY , AND SO ON.
C
C     FUEL AND HYDROGEN TRANSMITTANCES ( SEE TABLE 7-4 )
C
C     TC1  = ZETA 1 ( SEE EQ. 7-1 )
C     TC2  = ZETA 2 , AND SO ON ( SEE EQ. 7-1 ).
C     TP1  = EPSILON 1 ( SEE EQ. 7-2 )
C     TP2  = EPSILON 2 , AND SO ON ( SEE EQ. 7-2 ).
C     YP11 = SA-HYDROGEN-PATH XI FOR GROUP 1 ( SEE EQ. 7-3 )
C     YP12 = SA-HYDROGEN-PATH XI FOR GROUP 2 ( SEE EQ. 7-3 )
C     YP13 = SA-HYDROGEN-PATH XI FOR GROUP 3 ( SEE EQ. 7-3 )
C     YP14 = SA-HYDROGEN-PATH XI FOR GROUP 4 ( SEE EQ. 7-3 )
C     YCC1 = AB-FUEL-PATH XI FOR GROUP 1 ( SEE EQ. 7-4 )
C     YCC2 = AB-FUEL-PATH XI FOR GROUP 2 ( SEE EQ. 7-4 )
C     YCC3 = AB-FUEL-PATH XI FOR GROUP 3 ( SEE EQ. 7-4 )
C     YCC4 = AB-FUEL-PATH XI FOR GROUP 4 ( SEE EQ. 7-4 )
C     YP21 = BC-HYDROGEN-PATH XI FOR GROUP 1 ( SEE EQ. 7-5 )
C     YP22 = BC-HYDROGEN-PATH XI FOR GROUP 2 ( SEE EQ. 7-5 )
C     YP23 = BC-HYDROGEN-PATH XI FOR GROUP 3 ( SEE EQ. 7-5 )
C     YP24 = BC-HYDROGEN-PATH XI FOR GROUP 4 ( SEE EQ. 7-5 )
C     XPP1 = SQ-HYDROGEN-PATH XI FOR GROUP 1 ( SEE EQ. 7-6 )
C     XPP2 = SQ-HYDROGEN-PATH XI FOR GROUP 2 ( SEE EQ. 7-6 )
C     XPP3 = SQ-HYDROGEN-PATH XI FOR GROUP 3 ( SEE EQ. 7-6 )
C     XPP4 = SQ-HYDROGEN-PATH XI FOR GROUP 4 ( SEE EQ. 7-6 )
C     EPP1 = SF-HYDROGEN-PATH ETA FOR GROUP 1 ( SEE EQ. 7-7 )
C     EPP2 = SF-HYDROGEN-PATH ETA FOR GROUP 2 , AND SO ON
C            ( SEE EQ. 7-7 ).
C     EF1  = FG-FUEL-PATH ETA FOR GROUP 1 ( SEE EQ. 7-8 )
C     EF2  = FG-FUEL-PATH ETA FOR GROUP 2 , AND SO ON
C            ( SEE EQ. 7-8 ).
C     OPP1 = SL-HYDROGEN-PATH FOR GROUP 1 ( SEE EQ. 7-10 )
C     OPP2 = SL-HYDROGEN-PATH FOR GROUP 2 , AND SO ON
C            ( SEE EQ. 7-10 ).
C     NOTE : SF-HYDROGEN-PATH TRANSMITTANCES EQUAL TO
C            GH-HYDROGEN-PATH TRANSMITTANCES ( SEE EQ. 7-9 AND
C            EQ. 7-7 ).
C
C     NEUTRON ABSORPTION BY THE FUEL REGION
C
C     Q530 = FRACTION OF FISSION NEUTRONS ( NEVER TRAVERSE THE
C            FUEL BOUNDARY ) WHICH ARE ABSORBED AS GROUP 1
C     Q531 = FRACTION OF FISSION NEUTRONS ( NEVER TRAVERSE THE
C            FUEL BOUNDARY ) WHICH ARE ABSORBED AS GROUP 2
C     Q532 = FRACTION OF FISSION NEUTRONS ( NEVER TRAVERSE THE
C            FUEL BOUNDARY ) WHICH ARE ABSORBED AS GROUP 3
C     Q533 = FRACTION OF FISSION NEUTRONS ( NEVER TRAVERSE THE
C            FUEL BOUNDARY ) WHICH ARE ABSORBED AS GROUP 4
C     Q534 = FRACTION OF FISSION NEUTRONS ( TRAVERSE THE FUEL
C            BOUNDARY ) WHICH ARE ABSORBED AS GROUP 1
C     Q535 = FRACTION OF FISSION NEUTRONS ( TRAVERSE THE FUEL
C            BOUNDARY ) WHICH ARE ABSORBED AS GROUP 2
C     Q536 = FRACTION OF FISSION NEUTRONS ( TRAVERSE THE FUEL
C            BOUNDARY ) WHICH ARE ABSORBED AS GROUP 3
C     Q537 = FRACTION OF FISSION NEUTRONS ( TRAVERSE THE FUEL
```

```
C            BOUNDARY ) WHICH ARE ABSORBED AS GROUP 4
C
C     NEUTRON ABSORPTION BY THE HYDROGEN REGION
C
C     Q330 = FRACTION OF FISSION NEUTRONS WHICH ARE ABSORBED
C            AS GROUP 1
C     Q331 = FRACTION OF FISSION NEUTRONS WHICH ARE ABSORBED
C            AS GROUP 2
C     Q332 = FRACTION OF FISSION NEUTRONS WHICH ARE ABSORBED
C            AS GROUP 3
C     Q333 = FRACTION OF FISSION NEUTRONS WHICH ARE ABSORBED
C            AS GROUP 4
C
C     EFFECTIVE NEUTRON MULTIPLICATION FACTOR
C
C     EKEF = EFFECTIVE NEUTRON MUTIPLICATION FACTOR
C     **********************************************************
C
      WRITE(02,100)RP,RC
      WRITE(02,101)PDF,RDEL,AF
      WRITE(02,102)RA11,RA12,RA13,RA14
      WRITE(02,103)RA22,RA23,RA24
      WRITE(02,104)RA33,RA34,RA44
      WRITE(02,105)TC1,TC2,TC3,TC4
      WRITE(02,106)TP1,TP2,TP3,TP4
      WRITE(02,107)YP11,YP12,YP13,YP14
      WRITE(02,108)YCC1,YCC2,YCC3,YCC4
      WRITE(02,109)YP21,YP22,YP23,YP24
      WRITE(02,110)XPP1,XPP2,XPP3,XPP4
      WRITE(02,111)EPP1,EPP2,EPP3,EPP4
      WRITE(02,112)EF1,EF2,EF3,EF4
      WRITE(02,113)OPP1,OPP2,OPP3,OPP4
      WRITE(02,114)Q530,Q531,Q532,Q533
      WRITE(02,115)Q534,Q535,Q536,Q537
      WRITE(02,116)Q330,Q331,Q332,Q333
      WRITE(02,117)EKEF
 100  FORMAT(3X,'RP  =',E12.5,1X,'RC  =',E12.5)
 101  FORMAT(3X,'PDF =',E12.5,1X,'RDEL=',E12.5,1X,'AF  =',E12.5)
 102  FORMAT(3X,'RA11=',E12.5,1X,'RA12=',E12.5,1X,'RA13=',E12.5,
     #1X,'RA14=',E12.5)
 103  FORMAT(3X,'RA22=',E12.5,1X,'RA23=',E12.5,1X,'RA24=',E12.5)
 104  FORMAT(3X,'RA33=',E12.5,1X,'RA34=',E12.5,1X,'RA44=',E12.5)
 105  FORMAT(3X,'TC1 =',E12.5,1X,'TC2 =',E12.5,1X,'TC3 =',E12.5,
     #1X,'TC4 =',E12.5)
 106  FORMAT(3X,'TP1 =',E12.5,1X,'TP2 =',E12.5,1X,'TP3 =',E12.5,
     #1X,'TP4 =',E12.5)
 107  FORMAT(3X,'YP11=',E12.5,1X,'YP12=',E12.5,1X,'YP13=',E12.5,
     #1X,'YP14=',E12.5)
 108  FORMAT(3X,'YCC1=',E12.5,1X,'YCC2=',E12.5,1X,'YCC3=',E12.5,
     #1X,'YCC4=',E12.5)
 109  FORMAT(3X,'YP21=',E12.5,1X,'YP22=',E12.5,1X,'YP23=',E12.5,
     #1X,'YP24=',E12.5)
 110  FORMAT(3X,'XPP1=',E12.5,1X,'XPP2=',E12.5,1X,'XPP3=',E12.5,
     #1X,'XPP4=',E12.5)
 111  FORMAT(3X,'EPP1=',E12.5,1X,'EPP2=',E12.5,1X,'EPP3=',E12.5,
     #1X,'EPP4=',E12.5)
 112  FORMAT(3X,'EF1 =',E12.5,1X,'EF2 =',E12.5,1X,'EF3 =',E12.5,
     #1X,'EF4 =',E12.5)
 113  FORMAT(3X,'OPP1=',E12.5,1X,'OPP2=',E12.5,1X,'OPP3=',E12.5,
     #1X,'OPP4=',E12.5)
 114  FORMAT(3X,'Q530=',E12.5,1X,'Q531=',E12.5,1X,'Q532=',E12.5,
     #1X,'Q533=',E12.5)
 115  FORMAT(3X,'Q534=',E12.5,1X,'Q535=',E12.5,1X,'Q536=',E12.5,
     #1X,'Q537=',E12.5)
 116  FORMAT(3X,'Q330=',E12.5,1X,'Q331=',E12.5,1X,'Q332=',E12.5,
     #1X,'Q333=',E12.5)
 117  FORMAT(3X,'EFFECTIVE MULTIPLICATION FACTOR',9X,'=',E12.5)
 999  STOP
      END
```

Source Listing of ALB5

```
C
C     ***********************************************************
C     REFLECTOR ALBEDOS FOR FINITE REFLECTOR
C     ( P1 - SPHERICAL GEOMETRY / FOUR - GROUP ENERGY )
C     ***********************************************************
C
      PROGRAM ALB5
      NTIN=01
      NTOUT=02
      OPEN(NTIN, FILE='ALB5.IN', STATUS='OLD')
      OPEN(NTOUT, FILE='ALB5.OUT', STATUS='UNKNOWN')
C
C     ***********************************************************
C                         INPUT DATA
C
C     R    = INNER RADIUS OF THE REFLECTOR
C     T    = THICKNESS OF THE REFLETOR
C
C     REFLECTOR GROUP PARAMETERS ( SEE TABLE 6-3 )
C
C     DR1  = DIFFUSION COEFFICIENT FOR GROUP 1
C     DR2  = DIFFUSION COEFFICIENT FOR GROUP 2 , AND SO ON.
C     ERN1 = MACROSCOPIC (n,2n) CROSS SECTION FOR GROUP 1
C     ERA1 = MACROSCOPIC ABSORPTION CROSS SECTION FOR GROUP 1
C     ERA2 = MACROSCOPIC ABSORPTION CROSS SECTION FOR GROUP 2 ,
C            AND SO ON.
C     ERS12= MACROSCOPIC GROUP-TRANSFER CROSS SECTION FROM
C            GROUP 1 TO GROUP 2
C     ERS13= MACROSCOPIC GROUP-TRANSFER CROSS SECTION FROM
C            GROUP 1 TO GROUP 3 , AND SO ON.
C     ***********************************************************
C
      READ(01,10)R,T
      READ(01,11)DR1,DR2,DR3,DR4,ERN1,ERA1,ERA2,ERA3,ERA4,
     #ERS12,ERS13,ERS14,ERS23,ERS24,ERS34
 10   FORMAT(2E17.7)
 11   FORMAT(4E17.7)
      ERR1=ERA1+ERS12+ERS13+ERS14-ERN1
      ERR2=ERA2+ERS23+ERS24
      ERR3=ERA3+ERS34
      ERR4=ERA4
      RK1=SQRT(ERR1/DR1)
      RK2=SQRT(ERR2/DR2)
      RK3=SQRT(ERR3/DR3)
      RK4=SQRT(ERR4/DR4)
C
C     ***********************************************************
C     REFLECTOR ALBEDO CALCULATION
C     ***********************************************************
C
      H=R+T
      X1=EXP(-RK1*R)
      X2=EXP(-RK2*R)
      X3=EXP(-RK3*R)
      X4=EXP(-RK4*R)
      X5=EXP(+RK1*R)
      X6=EXP(+RK2*R)
      X7=EXP(+RK3*R)
      X8=EXP(+RK4*R)
      X9=EXP(-RK1*H)
      X10=EXP(-RK2*H)
      X11=EXP(-RK3*H)
      X12=EXP(-RK4*H)
      X13=EXP(+RK1*H)
      X14=EXP(+RK2*H)
      X15=EXP(+RK3*H)
      X16=EXP(+RK4*H)
      X17=DR1*(RK1+1./H)/2.
      X18=DR1*(RK1-1./H)/2.
      X19=DR1*(RK1+1./R)/2.
      X20=DR1*(RK1-1./R)/2.
      X21=DR2*(RK1*RK1-RK2*RK2)
      X22=2.*DR2*(RK1+1./R)
      X23=2.*DR2*(RK1-1./R)
      X24=2.*DR2*(RK2+1./R)
```

```
 X25=2.*DR2*(RK2-1./R)
 X26=2.*DR2*(RK1+1./H)
 X27=2.*DR2*(RK1-1./H)
 X28=2.*DR2*(RK2+1./H)
 X29=2.*DR2*(RK2-1./H)
 X30=DR3*(RK1*RK1-RK3*RK3)
 X31=DR3*(RK2*RK2-RK3*RK3)
 X32=2.*DR3*(RK1+1./R)
 X33=2.*DR3*(RK1-1./R)
 X34=2.*DR3*(RK2+1./R)
 X35=2.*DR3*(RK2-1./R)
 X36=2.*DR3*(RK3+1./R)
 X37=2.*DR3*(RK3-1./R)
 X38=2.*DR3*(RK1+1./H)
 X39=2.*DR3*(RK1-1./H)
 X40=2.*DR3*(RK2+1./H)
 X41=2.*DR3*(RK2-1./H)
 X42=2.*DR3*(RK3+1./H)
 X43=2.*DR3*(RK3-1./H)
 X44=DR4*(RK1*RK1-RK4*RK4)
 X45=DR4*(RK2*RK2-RK4*RK4)
 X46=DR4*(RK3*RK3-RK4*RK4)
 X47=2.*DR4*(RK1+1./R)
 X48=2.*DR4*(RK1-1./R)
 X49=2.*DR4*(RK2+1./R)
 X50=2.*DR4*(RK2-1./R)
 X51=2.*DR4*(RK3+1./R)
 X52=2.*DR4*(RK3-1./R)
 X55=2.*DR4*(RK4+1./R)
 X56=2.*DR4*(RK4-1./R)
 X57=2.*DR4*(RK1+1./H)
 X58=2.*DR4*(RK1-1./H)
 X59=2.*DR4*(RK2+1./H)
 X60=2.*DR4*(RK2-1./H)
 X61=2.*DR4*(RK3+1./H)
 X62=2.*DR4*(RK3-1./H)
 X63=2.*DR4*(RK4+1./H)
 X64=2.*DR4*(RK4-1./H)
 Y11=X9*(.25-X17)/H
 Y12=X13*(.25+X18)/H
 Y13=X1*(.25+X19)/R
 Y14=X5*(.25-X20)/R
 YD1=Y11*Y14-Y12*Y13
 Y15=-Y12/YD1
 Y16=Y11/YD1
 RA11=Y15*X1*(.25-X19)/R+Y16*X5*(.25+X20)/R
 Y17=-ERS12*Y15/X21
 Y18=-ERS12*Y16/X21
 Y19=-Y17*X1*(1.+X22)-Y18*X5*(1.-X23)
 Y20=X2*(1.+X24)
 Y21=X6*(1.-X25)
 Y22=-Y17*X9*(1.-X26)-Y18*X13*(1.+X27)
 Y23=X10*(1.-X28)
 Y24=X14*(1.+X29)
 YD2=Y20*Y24-Y21*Y23
 Y25=(Y19*Y24-Y22*Y21)/YD2
 Y26=(Y20*Y22-Y19*Y23)/YD2
 Y27=(Y25*X2+Y26*X6+Y17*X1+Y18*X5)/R
 RA12=Y27/2.
 Y28=-(ERS13*Y15+ERS23*Y17)/X30
 Y29=-(ERS13*Y16+ERS23*Y18)/X30
 Y30=-ERS23*Y25/X31
 Y31=-ERS23*Y26/X31
 Y32=-Y28*X1*(1.+X32)-Y29*X5*(1.-X33)-Y30*X2*(1.+X34)-Y31*
#X6*(1.-X35)
 Y33=X3*(1.+X36)
 Y34=X7*(1.-X37)
 Y35=-Y28*X9*(1.-X38)-Y29*X13*(1.+X39)-Y30*X10*(1.-X40)-
#Y31*X14*(1.+X41)
 Y36=X11*(1.-X42)
 Y37=X15*(1.+X43)
 YD3=Y33*Y37-Y34*Y36
 Y38=(Y32*Y37-Y35*Y34)/YD3
 Y39=(Y33*Y35-Y32*Y36)/YD3
 Y40=(Y38*X3+Y39*X7+Y28*X1+Y29*X5+Y30*X2+Y31*X6)/R
 RA13=Y40/2.
 Y41=-(ERS14*Y15+ERS24*Y17+ERS34*Y28)/X44
 Y42=-(ERS14*Y16+ERS24*Y18+ERS34*Y29)/X44
 Y43=-(ERS24*Y25+ERS34*Y30)/X45
```

```
 Y44=-(ERS24*Y26+ERS34*Y31)/X45
 Y45=-ERS34*Y38/X46
 Y46=-ERS34*Y39/X46
 Y47=-Y41*X1*(1.+X47)-Y42*X5*(1.-X48)-Y43*X2*(1.+X49)-Y44*
#X6*(1.-X50)-Y45*X3*(1.+X51)-Y46*X7*(1.-X52)
 Y48=X4*(1.+X55)
 Y49=X8*(1.-X56)
 Y50=-Y41*X9*(1.-X57)-Y42*X13*(1.+X58)-Y43*X10*(1.-X59)-
#Y44*X14*(1.+X60)-Y45*X11*(1.-X61)-Y46*X15*(1.+X56)
 Y51=X12*(1.-X63)
 Y52=X16*(1.+X64)
 YD4=Y48*Y52-Y49*Y51
 Y53=(Y47*Y52-Y49*Y50)/YD4
 Y54=(Y48*Y50-Y47*Y51)/YD4
 Y55=(Y53*X4+Y54*X8+Y41*X1+Y42*X5+Y43*X2+Y44*X6+Y45*X3+Y46*
#X7)/R
 RA14=Y55/2.
 Y56=X10*(.25-X28/4.)/H
 Y57=X14*(.25+X29/4.)/H
 Y58=X2*(.25+X24/4.)/R
 Y59=X6*(.25-X25/4.)/R
 YD5=Y56*Y59-Y57*Y58
 Y60=-Y57/YD5
 Y61=Y56/YD5
 RA22=Y60*X2*(.25-X24/4.)/R+Y61*X6*(.25+X25/4.)/R
 Y62=-ERS23*Y60/X31
 Y63=-ERS23*Y61/X31
 Y64=-Y62*X2*(1.+X34)-Y63*X6*(1.-X35)
 Y65=X3*(1.+X36)
 Y66=X7*(1.-X37)
 Y67=-Y62*X10*(1.-X40)-Y63*X14*(1.+X41)
 Y68=X11*(1.-X42)
 Y69=X15*(1.+X43)
 YD6=Y65*Y69-Y66*Y68
 Y70=(Y64*Y69-Y66*Y67)/YD6
 Y71=(Y65*Y67-Y64*Y68)/YD6
 Y72=(Y70*X3+Y71*X7+Y62*X2+Y63*X6)/R
 RA23=Y72/2.
 Y73=-(ERS24*Y60+ERS34*Y62)/X45
 Y74=-(ERS24*Y61+ERS34*Y63)/X45
 Y75=-ERS34*Y70/X46
 Y76=-ERS34*Y71/X46
 Y77=-Y73*X2*(1.+X49)-Y74*X6*(1.-X50)-Y75*X3*(1.+X51)-Y76*
#X7*(1.-X52)
 Y78=X4*(1.+X55)
 Y79=X8*(1.-X56)
 Y80=-Y73*X10*(1.-X59)-Y74*X14*(1.+X60)-Y75*X11*(1.-X61)-
#Y76*X15*(1.+X62)
 Y81=X12*(1.-X63)
 Y82=X16*(1.+X64)
 YD7=Y78*Y82-Y79*Y81
 Y83=(Y77*Y82-Y79*Y80)/YD7
 Y84=(Y78*Y80-Y77*Y81)/YD7
 Y85=(Y83*X4+Y84*X8+Y73*X2+Y74*X6+Y75*X3+Y76*X7)/R
 RA24=Y85/2.
 Y86=X11*(.25-X42/4.)/H
 Y87=X15*(.25+X43/4.)/H
 Y88=X3*(.25+X36/4.)/R
 Y89=X7*(.25-X37/4.)/R
 YD8=Y86*Y89-Y87*Y88
 Y90=-Y87/YD8
 Y91=Y86/YD8
 RA33=Y90*X3*(.25-X36/4.)/R+Y91*X7*(.25+X37/4.)/R
 Y92=-ERS34*Y90/X46
 Y93=-ERS34*Y91/X46
 Y94=-Y92*X3*(1.+X51)-Y93*X7*(1.-X52)
 Y95=X4*(1.+X55)
 Y96=X8*(1.-X56)
 Y97=-Y92*X11*(1.-X61)-Y93*X15*(1.+X62)
 Y98=X12*(1.-X63)
 Y99=X16*(1.+X64)
 YD9=Y95*Y99-Y96*Y98
 Y100=(Y94*Y99-Y96*Y97)/YD9
 Y101=(Y95*Y97-Y94*Y98)/YD9
 Y102=(Y100*X4+Y101*X8+Y92*X3+Y93*X7)/R
 RA34=Y102/2.
 Y103=X12*(.25-X36/4.)/H
 Y104=X16*(.25+X64/4.)/H
 Y105=X4*(.25+X55/4.)/R
```

```
      Y106=X8*(.25-X56/4.)/R
      YD10=Y103*Y106-Y104*Y105
      Y107=-Y104/YD10
      Y108=Y103/YD10
      RA44=Y107*X4*(.25-X55/4.)/R+Y108*X8*(.25+X56/4.)/R
C
C     ***********************************************************
C                          OUTPUT RESULTS
C
C     REFLECTOR ALBEDOS
C
C     RA11 = PROBABILITY THAT A NEUTRON FROM GROUP 1 BE
C            REFLECTED WITH GROUP 1 ENERGY
C     RA12 = PROBABILITY THAT A NEUTRON FROM GROUP 1 BE
C            REFLECTED WITH GROUP 2 ENERGY , AND SO ON.
C     ***********************************************************
C
      WRITE(02,100)R,T
      WRITE(02,101)RA11,RA12,RA13,RA14
      WRITE(02,102)RA22,RA23,RA24
      WRITE(02,103)RA33,RA34,RA44
 100  FORMAT(3X,'R   =',E12.5,1X,'T   =',E12.5)
 101  FORMAT(3X,'RA11=',E12.5,1X,'RA12=',E12.5,1X,'RA13=',E12.5,
     #1X,'RA14=',E12.5)
 102  FORMAT(3X,'RA22=',E12.5,1X,'RA23=',E12.5,1X,'RA24=',E12.5)
 103  FORMAT(3X,'RA33=',E12.5,1X,'RA34=',E12.5,1X,'RA44=',E12.5)
      STOP
      END
```

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BIOGRAPHICAL SKETCH

Ronaldo Glicerio Cabral was born on July 8, 1946, in Rio de Janeiro, Brazil. In December of 1968, Ronaldo Glicerio Cabral graduated as an army officer at the Academia Militar de Agulhas Negras (Agulhas Negras Military Academy) in Rio de Janeiro, Brazil. In December of 1976, he received the degree of Bachelor of Science in electrical engineering from the Instituto Militar de Engenharia (Military Engineering Institute), in Rio de Janeiro, Brazil. He received the degree of Master of Science in nuclear engineering in December of 1981, from the same Institute.

Ronaldo Glicerio Cabral worked on many projects for the Brazilian Army and taught courses in the Nuclear Engineering Department in the Military Engineering Institute.

In January of 1989, he was admitted to the graduate program of the Nuclear Engineering Department at the University of Florida to pursue a Ph.D. degree, which has been sponsored by the Brazilian Army.

Ronaldo Glicerio Cabral has been a member of Alpha Nu Sigma, the Nuclear Engineering Honor Society, since 1990.

He is married to Aureliana Azambuja Costa Cabral. Their son, Ronaldo Glicerio Cabral Filho, is 16 years old.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Alan M. Jacobs

Alan M. Jacobs, Chairman
Professor of Nuclear
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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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This dissertation was submitted to the Graduate Faculty of the College of Engineering and to the Graduate School and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

December 1991

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